

# Will the Real Number of Epicycles Stand Up?



Or

What are they  
really teaching in  
Astronomy  
courses these  
days?

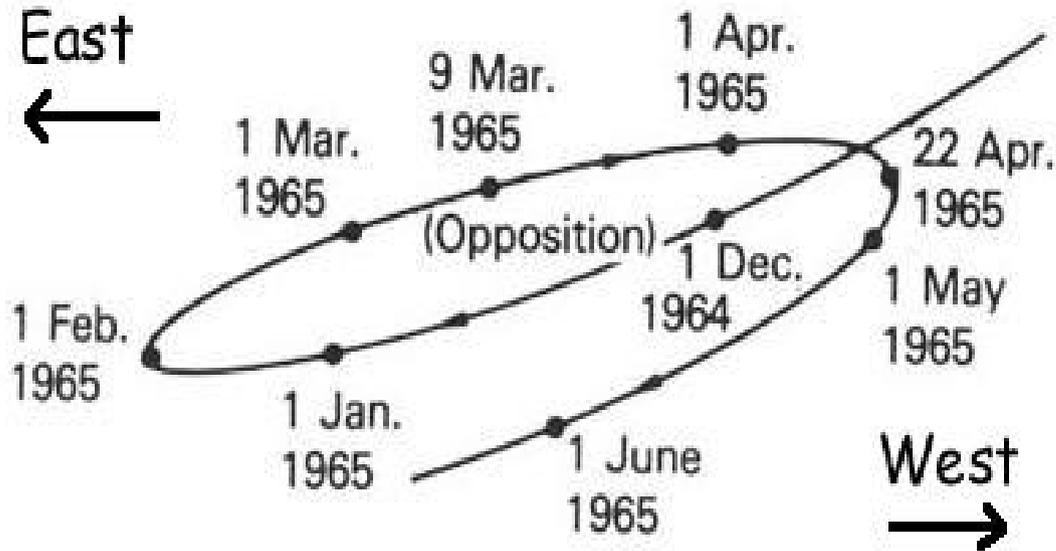
3<sup>rd</sup> International Conference on Absolutes  
Houston, Texas, July 16-18, 2007

# Motivation

“Perhaps no other subject in the history of science has had more distortions and misunderstandings than the development of ancient astronomy. For example, one of the most persistent canards is that Ptolemy's model of planetary motion required up to 80 epicycles which prompted the search for simpler models.”

Michael Nauenberg, University of California Santa Cruz  
<http://physics.ucsc.edu/~michael/longbeach.html>

# Why Epicycles?: Wandering Planets



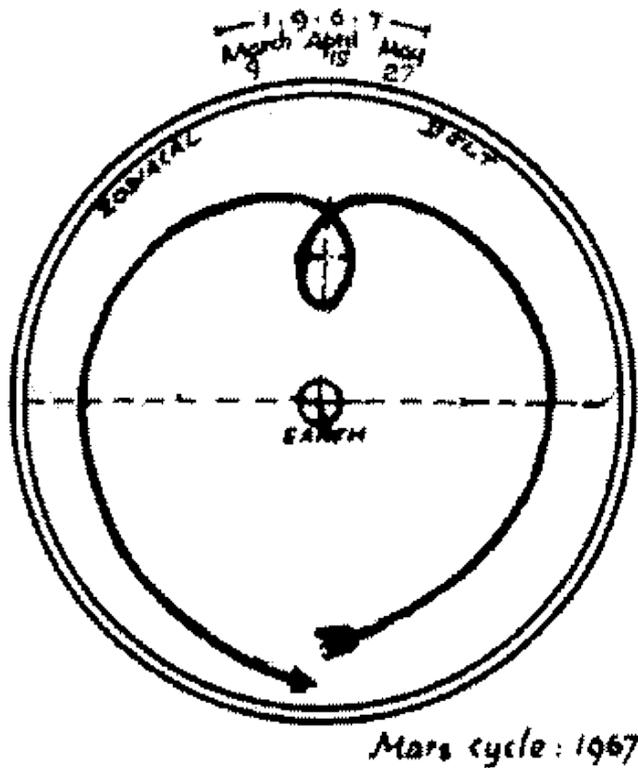
The planets are called wanderers (Greek: planetai) due to their retrograde motion.

Retrograde motion is when a planet begins to go backward (i.e. retro), and later on, and goes forward again.

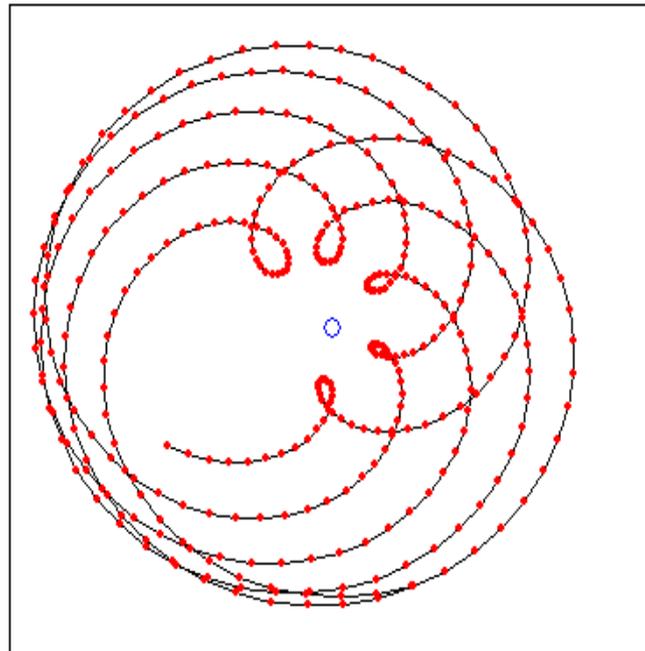
The *Sun*, *moon* and *stars* do not retrograde.

	Retrograde	Length	Orbit Revolution
Mercury	4 months	20-24 days	2.93 months (87.97 days)
Venus	18 months	40-43 days	7.49 months (224.7 days)
Mars	25.7 months	60-81 days	1.88 years
Jupiter		120 days	11.86 years
Saturn		130 days	29.46 years

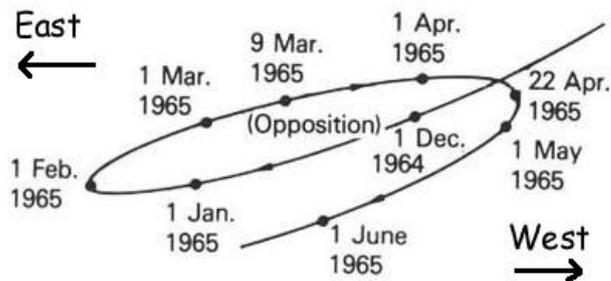
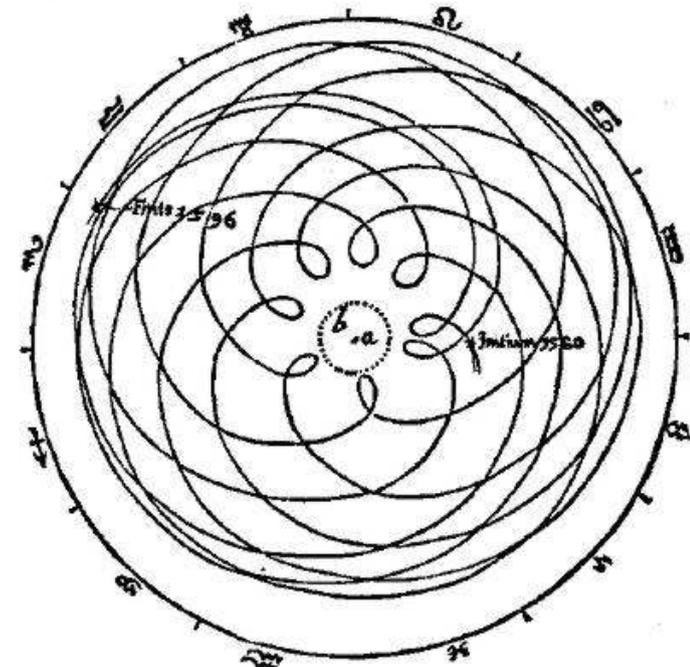
# Wandering Mars



Mars: 2001-2010



DE MOTIB. STELLÆ MARTIS

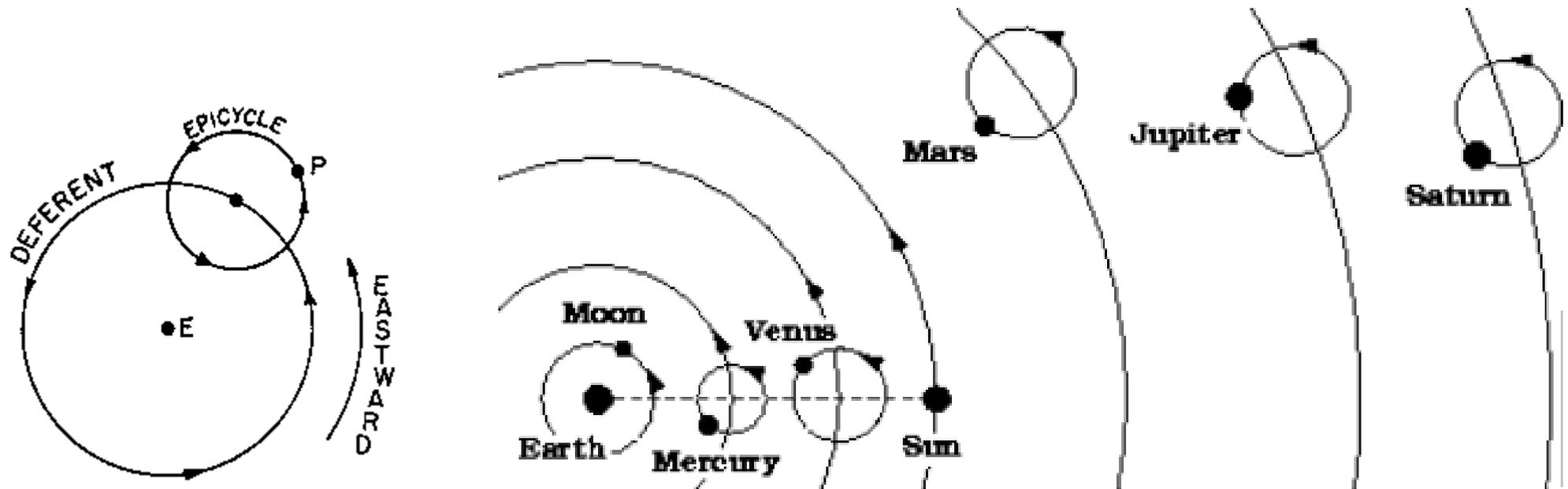


Mars wanders (retrogrades) every 25.7 months with a length of 60 to 81 days.

Ptolemy (150AD) by using the Equant was the first Greek to mathematically fit Mars wandering motion.

# The number of Ptolemy's Epicycles

The number of *longitudinal* epicycles for Ptolemy are typically shown as this:



Total epicycles are Moon(0), Mercury(1), Venus(1), Sun(0), Mars(1), Jupiter(1), Saturn(1). Depending on how one counts, it looks like 7 deferents, plus 5 *epicycles* = 12 circles.

“A comparison between the Copernican and the classical Ptolemaic system is more precise if we limit the count of circles to the longitude mechanisms for the Sun (or Earth), Moon, and planets: *Copernicus requires 18, Ptolemy requires 15.*” *The Eye of Heaven*, page 197.



# 40, 80, 80+, heaps of Epicycles (2007)

But to accurately account for the planets motions, the epicycles need epicycles!  
Eventually the Ptolemaic model needed 16 levels of epicycles, plus off-center orbits for the Sun and planets...

Astr 201 and Astr 221, Prof. Chris Mihos, Case Western Reserve University,  
<http://burro.astr.cwru.edu/Academics/Astr221/Gravity/history.htm>

“48 epicycles, compared to 40 in the Ptolemaic geocentric system”

Astronomy 161: An Introduction to Solar System Astronomy  
Prof. Richard Pogge, Professor of Astronomy  
The Ohio State University, Department of Astronomy  
<http://www.astronomy.ohio-state.edu/~pogge/Ast161/Unit3/copernicus.html>

“This ultimately required 80 circles and nested epicycles, which was extremely complex and unwieldy”

ASTR 111: Introduction to Modern Astronomy I,  
Peter A. Becker, Professor of Astrophysical,  
Planetary, and Space Sciences, George Mason University.  
[http://www.scs.gmu.edu/~pbecker/lecture4\\_handouts.pdf](http://www.scs.gmu.edu/~pbecker/lecture4_handouts.pdf)

The system was very cumbersome with all those 80 arbitrary numbers needed to describe it.

Astronomy 101: Introduction to Astronomy,  
Dr. Kathleen Flint, Center from the National Science Foundation & Stony Brook Southampton  
<http://ms.cc.sunysb.edu/~kflint/ast101/PDFs/lecture3.pdf>

“This resulted in a horribly complex model, replete with epicycles, heaped upon deferents, upon equants, upon eccentrics.”

# 240 Epicycles

“Although Copernicus introduced the heliocentric theory of the solar system, he still found it necessary to introduce about 40 epicycles to account for observations, but he considered this a great improvement since the Ptolmaic theory contained more than 240 such epicycles.”

*Lloyd Motz and Anneta Duveen, Essentials of Astronomy, Wadsworth Publishing Company, Inc., Belmont, California, 1966, page 135, (Preface says, “This book is the expanded version of a two-semester introductory course in astronomy given at Columbia University over a period of years.”)*

Lloyd Motz (1909-2004) was an American astronomer. earned a Ph.D. in physics from Columbia University in 1936. He was the author of 21 books on astronomy. Motz was noted for having defeated Enrico Fermi in a tennis match.

*So there you have it, a Professor at Columbia: teaching bad science!*

# Case School of Applied Science, 1880

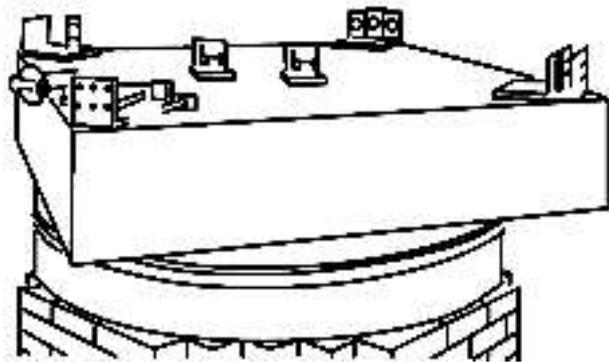
“But to accurately account for the planets motions, the epicycles need epicycles! Eventually the Ptolemaic model needed 16 levels of epicycles, plus off-center orbits for the Sun and planets.”

Astr201 and Astr 221, Prof. Chris Mihos,

Professor of Astronomy, Case Western Reserve University,

<http://burro.astr.cwru.edu/Academics/Astr201/Discovering/sky.html>

<http://burro.astr.cwru.edu/Academics/Astr221/Gravity/history.htm>



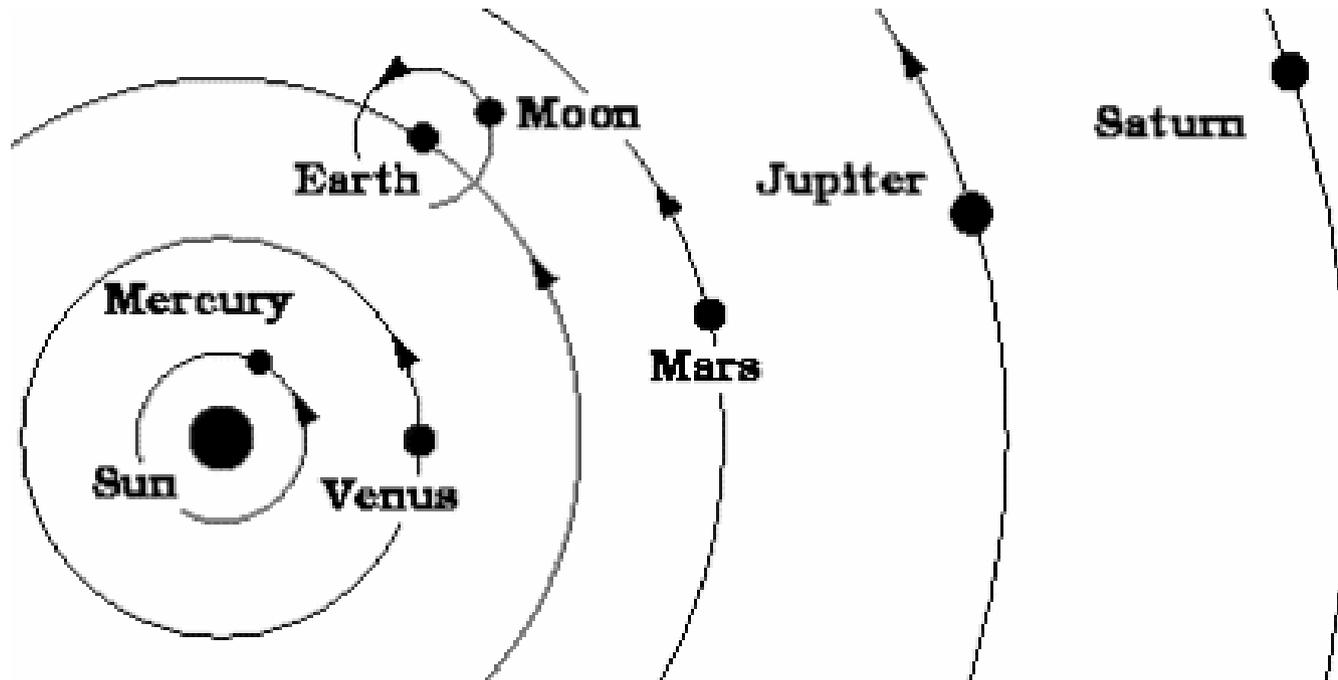
“We all know how badly science is taught in many schools.”

Professor Lawrence M. Krauss, Professor of Physics, Professor of Astronomy, and Director, *Center for Education and Research in Cosmology and Astrophysics* at Case Western Reserve University, July 26, 2004, Scientific American interview.

"It was you [Michelson-Morley experiment of 1887] who led physicists into new path, and through your marvelous experiment work paved the way for the development of the theory of relativity." Albert Einstein.

# Copernicus' 34 Epicycles

Astronomy textbooks deceptively show Copernicus' Model as nice and simple, including one epicycle!



But in the Copernicus' own words, in the *Commentariolus*, ~1512,

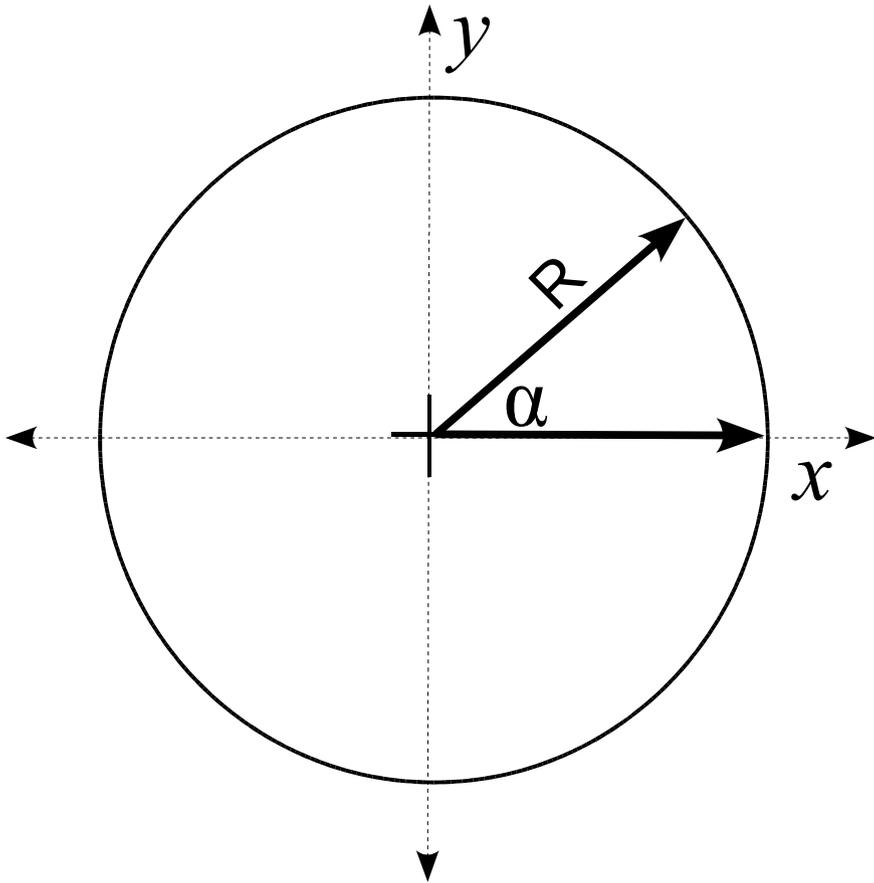
“Then Mercury runs on seven circles in all; Venus on five; the earth on three, and round it the moon on four; finally Mars, Jupiter, and Saturn on five each. Altogether, therefore, *thirty-four circles* suffice to explain the entire structure of the universe and entire ballet of the planets.”

# Copernicus' 34 Circles (not epicycles)

In Arthur Koestler's book, The Sleepwalkers, page 579, he states,

	<u>De Revolutionibus, 1543</u>	<u>Commentariolus, ~1512</u>
Earth Diurnal motion	1	1
Motions in longitude	3	1
Conic motion of axis	1	1
Fluctuations in rate of procession	4	0
Moon		
Motions in longitude	3	3
Motion in latitude	1	1
Three outer planets		
Motions in longitude (=3x3)	9	9
Oscillations in latitude (=2x3)	6	6
Venus		
Motions in longitude	3	3
Oscillations in latitude	6	2
Mercury		
Motions in longitude	5	5
Oscillations in latitude	6	2
<hr/> Total	<hr/> 48	<hr/> 34

# Pythagoreans' Circle



The Cartesian equation of a circle is

$$x^2 + y^2 = R^2$$

where  $R$  is the radius,  $\alpha$  is the angle

The parametric equation of a circle is

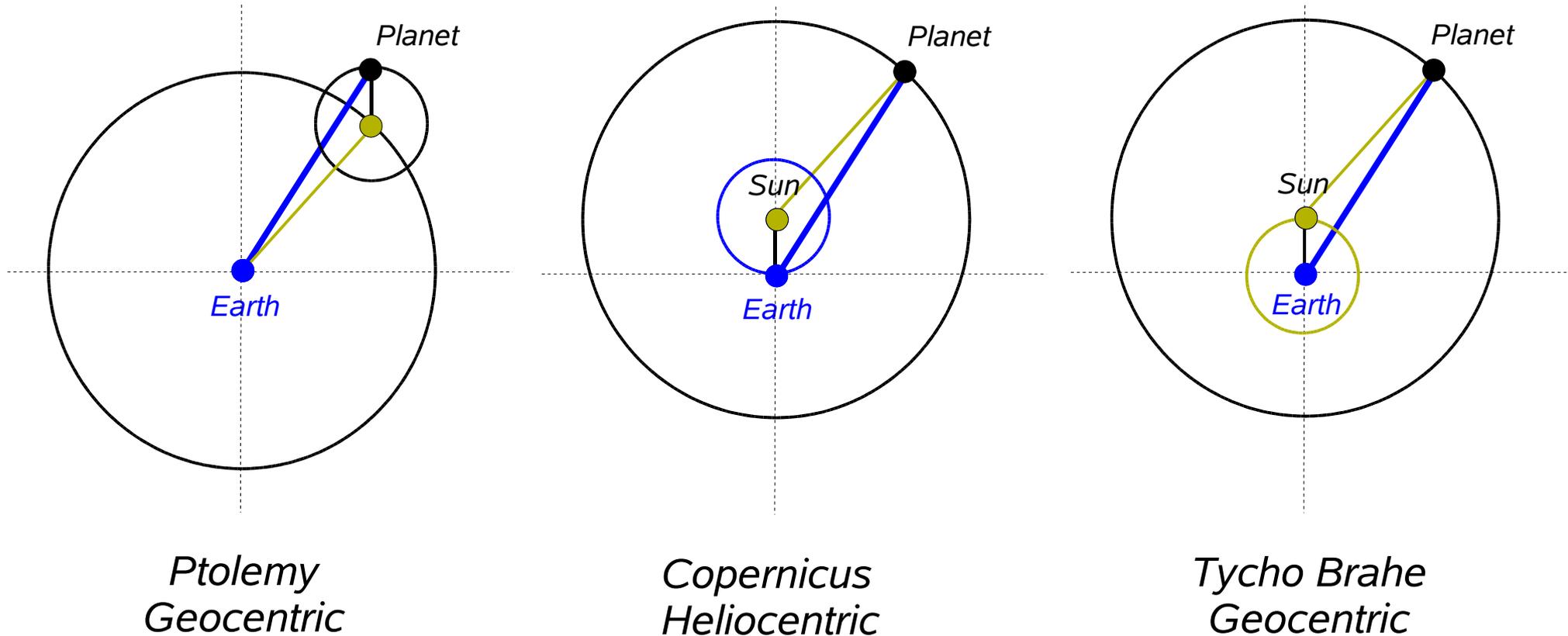
$$x = R \cos \alpha \quad \text{and} \quad y = R \sin \alpha$$

$$z = R \cos \alpha + i R \sin \alpha$$

Plato (c. 427-347 BC) based his cosmology from the Pythagoreans:

- The heavens are divine, and eternal: characterized by circular motion.
- The earth (material world) is corrupt: characterized by broken circle (rectilinear motion).

# Observationally Indistinguishable Models



These three models are observationally indistinguishable, including retrograde motion, page 21, *“It all started with Copernicus,”* Howard Margolis, 2002.

# Ptolemaic versus Copernican

Today, we cannot say that the Copernican theory is “right” and the Ptolemaic theory “wrong” in any meaningful physical sense. The two theories, when improved by adding terms involving the square of higher powers of the eccentricities of the planetary orbits, are physically equivalent to one another.”

Sir Fred Hoyle, page 88, “*Nicolaus Copernicus*,” Harper & Row, Publishers, 1973.

The late Sir Fred Hoyle (1915-2001), a world-renowned astronomer, is acknowledged to have been one of the most creative scientists of the 20th century.

*So, there you have this amazing statement. After 400 hundred years of promoting Heliocentric education (including parallax), the astronomers cannot honestly prove it.*

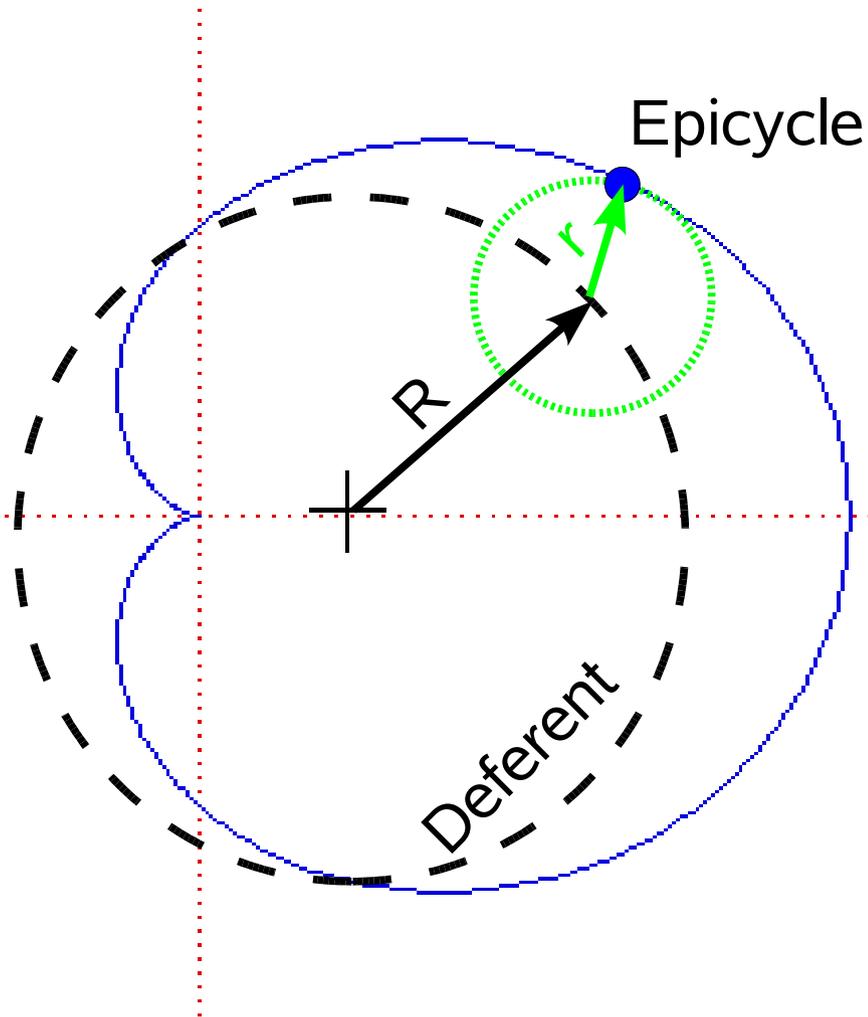
By the way, Fred Hoyle was no friend of the Bible,

“The passionate frenzy with which the Big Bang cosmology is clutched to the corporate scientific bosom evidently from a deep-rooted attachment to the first page of Genesis, religious fundamentalism at its strongest.”,

page 439, Big Bang by Simon Singh.

# Cardioid derived from the Epicycle

## Cardioid



The Epicycle parametric equation

$$x = R \cos \alpha + r \cos \beta$$

$$y = R \sin \alpha + r \sin \beta$$

The Cardioid curve

Let  $R =$  twice of the radius  $r = 2a$ ,

$$r = -a$$

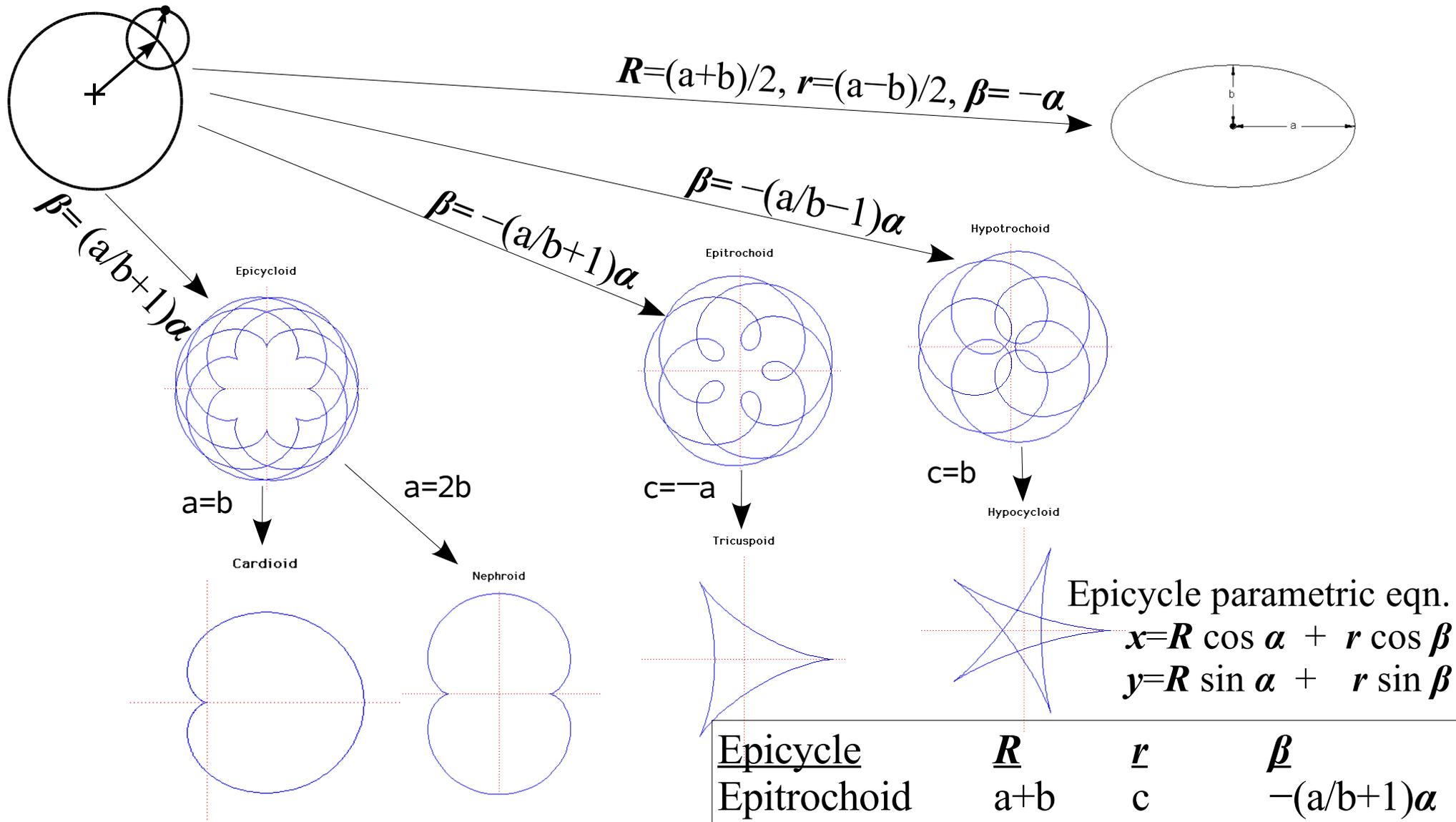
$\beta =$  twice the speed of  $\alpha = 2\alpha$

$$x = 2a \cos \alpha - a \cos 2\alpha$$

$$y = 2a \sin \alpha - a \sin 2\alpha$$

	$R$	$r$	$\beta$
Cardioid	$2a$	$-a$	$2\alpha$

# Famous curves derived from the Epicycle



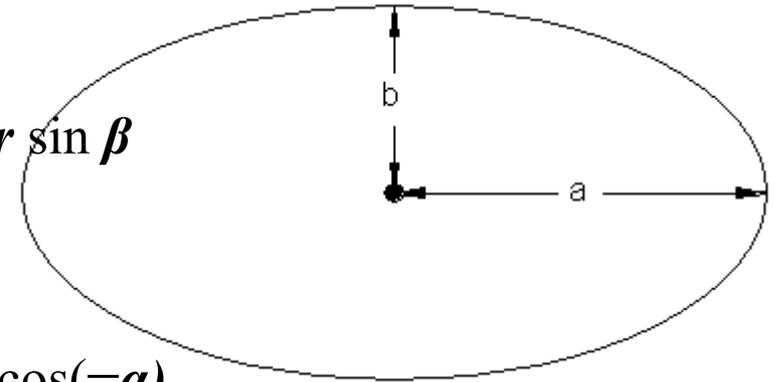
<u>Epicycle</u>	<u>R</u>	<u>r</u>	<u><math>\beta</math></u>
Epicycloid	$a+b$	$-b$	$(a/b+1)\alpha$
Cardioid	$2a$	$-a$	$2\alpha$
Nephroid	$3a$	$-a$	$2\alpha$

<u>Epicycle</u>	<u>R</u>	<u>r</u>	<u><math>\beta</math></u>
Epitrochoid	$a+b$	$c$	$-(a/b+1)\alpha$
Tricuspid	$2a$	$-a$	$-2\alpha$
Hypotrochoid	$a-b$	$c$	$-(a/b-1)\alpha$
Hypocycloid	$a-b$	$b$	$-(a/b-1)\alpha$

# Ellipse as an Epicycle

Astronomy textbooks conveniently ignore the fact the epicycle can describe an ellipse.

If we let  $R=(a+b)/2$ ,  $r=(a-b)/2$  and  $\beta = -\alpha$  then  
 an epicycle,  $x=R \cos \alpha + r \cos \beta$ ;  $y=R \sin \alpha + r \sin \beta$   
 will describe an ellipse,  $x = a \cos \theta$ ;  $y = b \sin \theta$



**Proof:**  $x=R \cos \alpha + r \cos \beta$   
 $\Rightarrow (a+b)/2 \cos(\alpha) + (a-b)/2 \cos(-\alpha)$   
 $\Rightarrow a/2 \cos(\alpha) + b/2 \cos(\alpha) + \underline{a/2 \cos(-\alpha)} - \underline{b/2 \cos(-\alpha)}$

And since *cosine* is an **even** function,  $\cos(-\alpha) \Rightarrow \cos(\alpha)$   
 $\Rightarrow a/2 \cos(\alpha) + b/2 \cos(\alpha) + a/2 \cos(\alpha) - b/2 \cos(\alpha)$   
 $\Rightarrow a/2 \cos(\alpha) + a/2 \cos(\alpha) \Rightarrow (a/2+a/2) \cos(\alpha) \Rightarrow a \cos(\alpha)$

$y=R \sin \alpha + r \sin \beta \Rightarrow (a+b)/2 \sin(\alpha) + (a-b)/2 \sin(-\alpha)$   
 $\Rightarrow a/2 \sin(\alpha) + b/2 \sin(\alpha) + \underline{a/2 \sin(-\alpha)} - \underline{b/2 \sin(-\alpha)}$

And since *sine* is an **odd** function,  $\sin(-\alpha) \Rightarrow -\sin(\alpha)$   
 $\Rightarrow a/2 \sin(\alpha) + b/2 \sin(\alpha) - a/2 \sin(\alpha) + b/2 \sin(\alpha)$   
 $\Rightarrow b/2 \sin(\alpha) + b/2 \sin(\alpha) \Rightarrow (b/2+b/2) \sin(\alpha) \Rightarrow b \sin(\alpha)$

# Multilevel Epicycles: Fourier Series

First, we change the epicycle to a series type notation:

Let  $\alpha = \omega_0 t$  and  $\beta = \omega_1 t$  where  $a_0, a_1, \omega_0, \omega_1$  are constants

$$x = R \cos \alpha + r \cos \beta \Rightarrow a_0 \cos \omega_0 t + a_1 \cos \omega_1 t$$

$$y = R \sin \alpha + r \sin \beta \Rightarrow a_0 \sin \omega_0 t + a_1 \sin \omega_1 t$$

Then, change it to complex notation,

$$z(t) = (a_0 \cos \omega_0 t + a_1 \cos \omega_1 t) + i(a_0 \sin \omega_0 t + a_1 \sin \omega_1 t)$$

$$z(t) = a_0 e^{i\omega_0 t} + a_1 e^{i\omega_1 t}$$

And if we now extend the terms of the epicycle to a multilevel series, clearly, it has the equivalent expressive power of a fourier series:

$$z(t) = a_0 e^{i\omega_0 t} + a_1 e^{i\omega_1 t} + a_2 e^{i\omega_2 t} + \dots$$

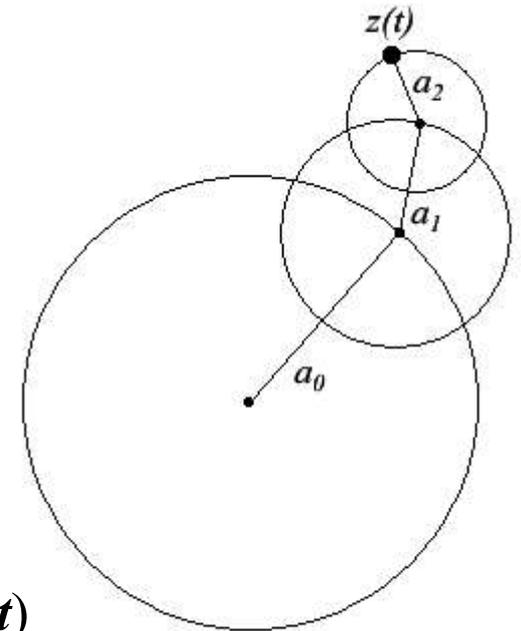
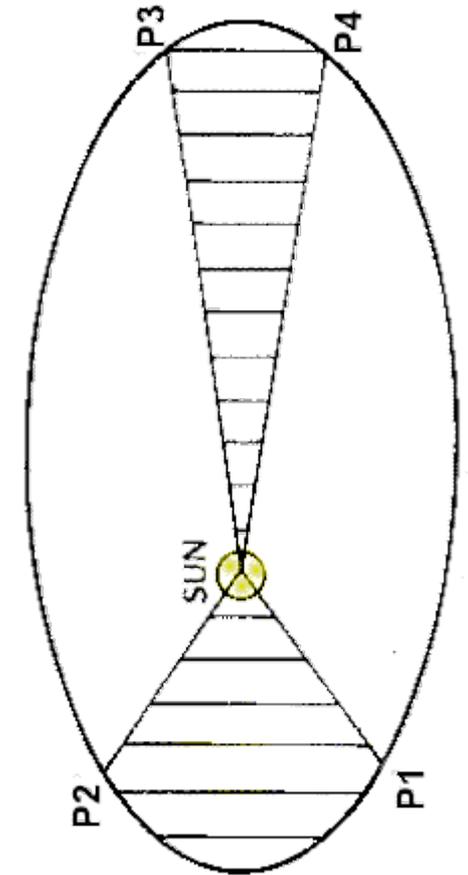
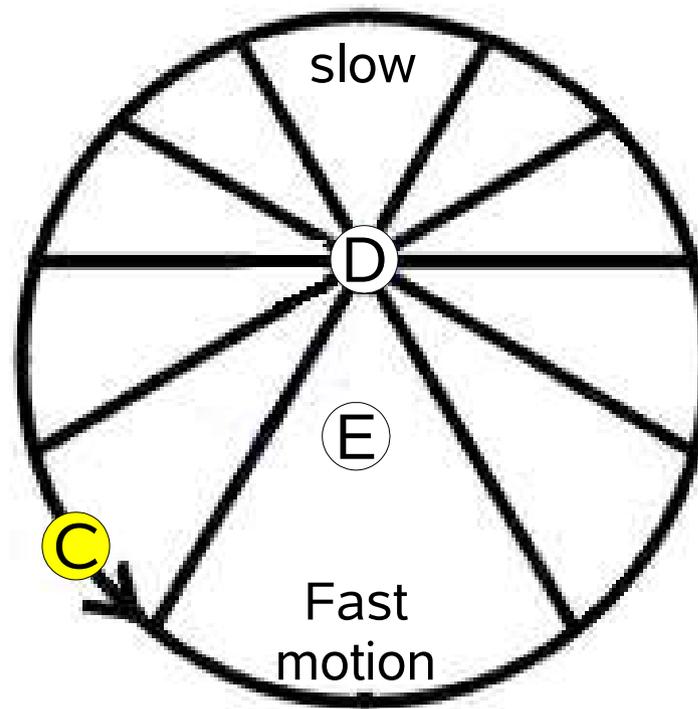
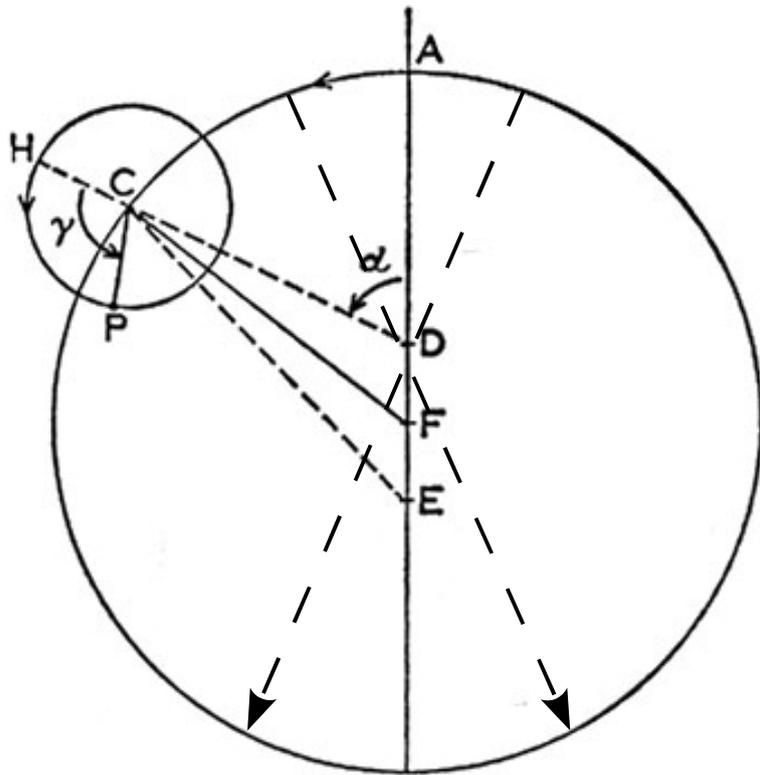


Figure: Series of Epicycles.

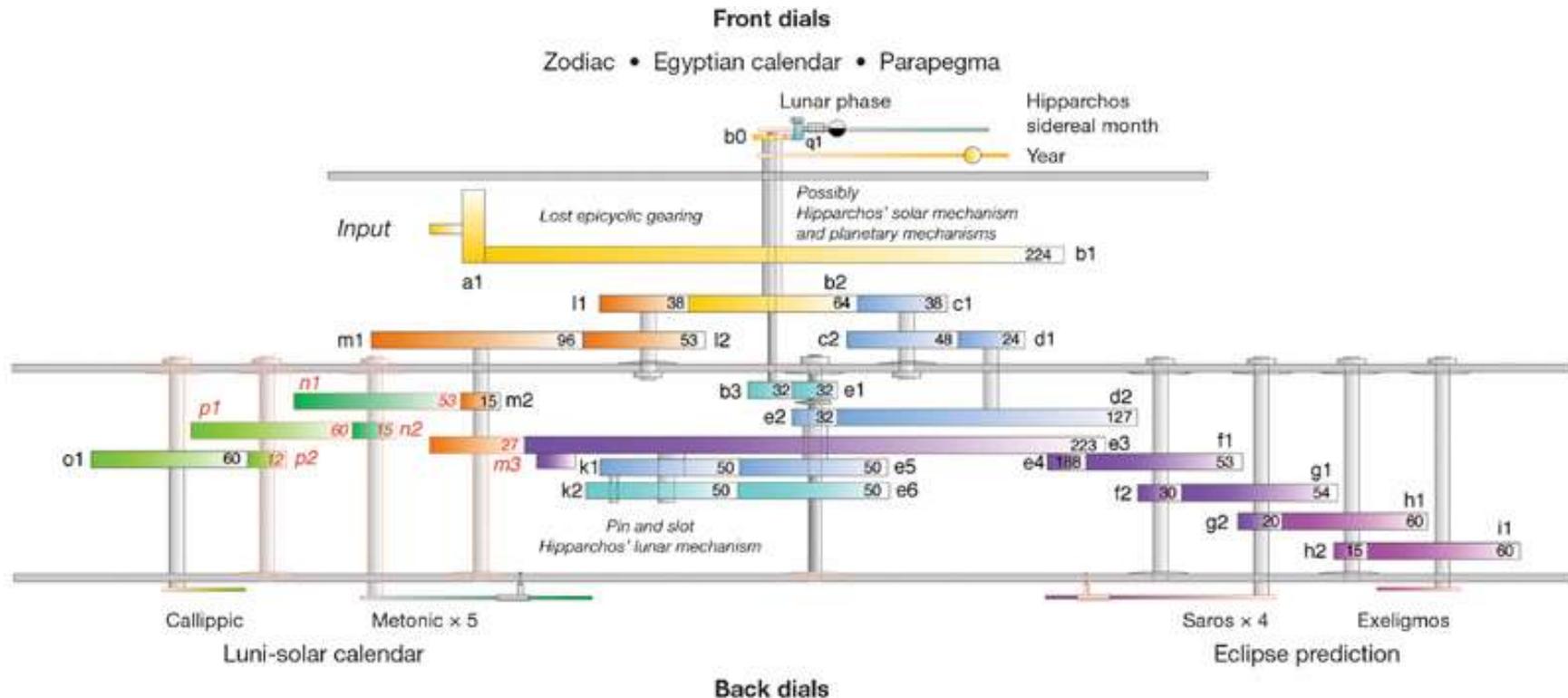
# Equant and Keplers 2<sup>nd</sup> Law

Ptolemy's *equant*, *D*, which is an extension to the epicycle, which adds *non-uniform* motion to the planet, *E*, approximately sweeps areas as Kepler's ellipse does, and also implies Ptolemy was well aware of Kepler's 2<sup>nd</sup> law :



# Sophistication of Greek Astronomy

“The Antikythera Mechanism is a unique Greek geared device, constructed around the end of the 2<sup>nd</sup> century BC. It is known that it calculated and displayed celestial information, particularly cycles such as the phases of the moon and a luni-solar calendar.”



“Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism,”

T. Freeth, Y. Bitsakis, et al., Nature 444, 587-591(30 November 2006)

<http://www.antikythera-mechanism.gr/>

<http://www.nature.com/nature/journal/v444/n7119/abs/nature05357.html>

# Textbooks: The Almagest, 1252

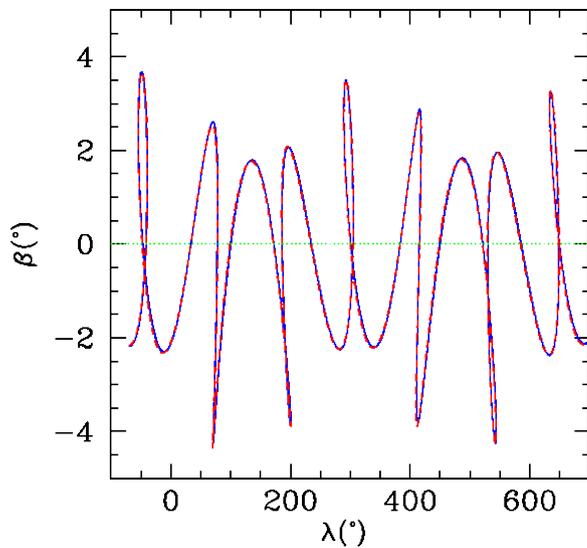
The Almagest consists of thirteen books (Euclid's Elements also contain 13 books). Their subject matter can be summarized as follows:

1. Overview of Aristotelian cosmology and spherical trigonometry.
2. Daily motion attributed to the heavens, and the length of daylight.
3. Motion of the Sun.
- 4-5. Motion of the Moon, lunar parallax, and the sizes and distances of the Sun and Moon relative to the Earth.
6. Solar and lunar eclipses.
- 7-8. Motions of the fixed stars, including precession of the equinoxes and a star catalogue.
9. Models for the five naked eye planets, as well as the motion of Mercury.
10. Motions of Venus and Mars.
11. Motions of Jupiter and Saturn.
12. Stations and retrogradations.
13. Motion in latitude of planets and Sun from the ecliptic.

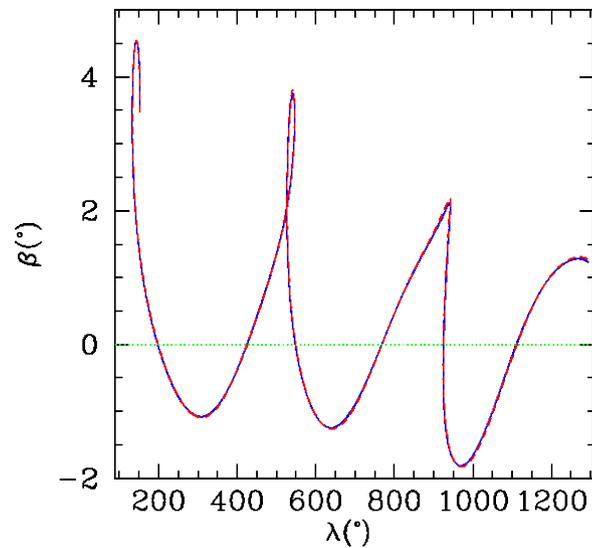
“The major post-Copernican ephemeris still based on the *Alfonsine Tables...*”, page 231, *The Eye of Heaven*, Owen Gingerich, The *Alfonsine Tables* followed Ptolemy very closely and did not add any more epicycles to the Ptolemy model.

# A Modern Almagest: Richard Fitzpatrick

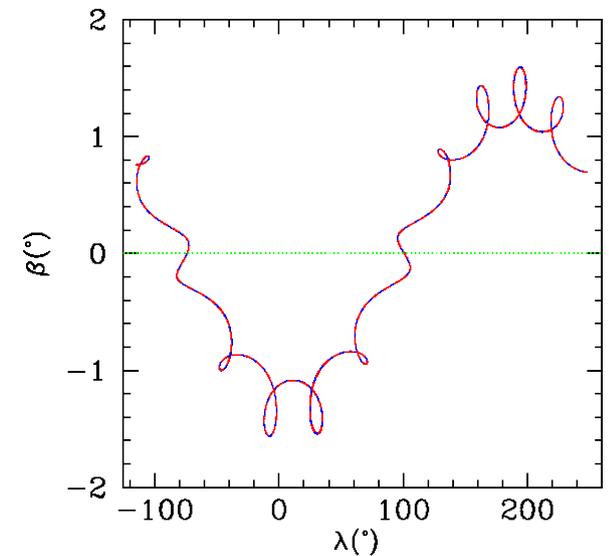
Prof. Fitzpatrick did research comparing Ptolemy's model by using modern parameters of 1995 and 1996. Shown below The ecliptic latitude of Mercury, Mars Jupiter versus its ecliptic longitude. The blue and red curves indicate the prediction of the updated Almagest model, and the original data, respectively.



Mercury



Mars



Jupiter

# A Modern Almagest: Conclusions

Prof. Fitzpatrick in his conclusion states,

“A number of important points emerge from our examination of Ptolemy's model of the Sun and the planets.”

“First, it is an excellent approximation to represent planetary orbits as eccentric circles.”

“Hence, Ptolemy's use of eccentric circles to represent planetary orbits in the Almagest is not a major source of error, as is commonly supposed.”

“Fourth, after the slight errors in Ptolemy's model are corrected, it reduces to a remarkably simple system consisting of just *nine circles* (four deferents and five epicycles).”

“Moreover, this simple system is able to account for the motion of the Sun and the planets visible to the naked eye to an accuracy which is more than adequate for naked-eye observations.”

***“The popular myth that Ptolemy's model requires a plethora of epicycles in order to accurately represent the observational data is simply untrue.”***

# References

## Recommended Popular Reading Books

Owen Gingerich, “The Book Nobody Read,” 2004, Penguin Books.

Fred Hoyle, “Nicolaus Copernicus,” 1973, Harper & Row Publishers.

Gerardus D. Bouw, “Geocentricity,” 1992, Cleveland, Ohio.

Owen Gingerich, “The Eye of Heaven Ptolemy, Copernicus, Kepler,” 1993, American Institute of Physics, New York.

Arthur Koestler, “The Sleepwalkers,” 1959, Arkana Penguin Books, reprint 1989.

Howard Margolis, “It all started with Copernicus,” McGraw-Hill, 2002.

## Serious Academic Books

Jose Chabas and Bernard R. Goldstein, “The Alfonsine Tables of Toledo,” 2003, Kluwer Academic Publishers.

C. M. Linton, “From Eudoxus to Einstein, A History of Mathematical Astronomy,” 2004, Cambridge University Press, UK.

Thomas S. Kuhn, “The Copernican Revolution, 1957, Harvard University Press, Cambridge, Mass.

Edward Rosen, “Three Copernican Treatises,” 1959, 2004, Dover Publications.

Sir Thomas L. Heath, “Greek Astronomy,” 1932, 1991, Dover Publications.

Scientific American interview, July 26, 2004, “Questions That Plague Physics: A Conversation with Lawrence M. Krauss,” [http://www.sciam.com/print\\_version.cfm?articleID=0009973A-D518-10FA-89FB83414B7F0000](http://www.sciam.com/print_version.cfm?articleID=0009973A-D518-10FA-89FB83414B7F0000)

# Appendix: More Quotes

“During this time the geocentric model of Ptolemy was still used but the Arab scholars found it necessary to continue to modify the model adding epicycle within epicycle. At one point the geocentric model had as many as 80 epicycles within epicycles. Quite ridiculous!”  
Astronomy 10, Introduction to General Astronomy, Bryan J. Méndez, Ph.D., University of California, Berkeley,  
<http://cse.ssl.berkeley.edu/bmendez/ay10/2002/notes/lec6.html>

“At the hands of the Arabs, Ptolemy's model was perfected. The Arabs also observed the sky and made ever more accurate measurements of the positions of celestial objects. The predictions of the original Ptolemaic model were accurate enough for the precision of observations of his time, but the new positions obtained with the accuracies achieved by the Arabs did not match the predictions. The Arab astronomers then, still confined to use circles by their aesthetic principles, incorporated epicycles within epicycles and off-centered orbits in order to "correct" the model.” Astronomy 100, Indiana University Northwest,  
Lucia M. Franco, Candidate, Ph.D., Astronomy & Astrophysics Dept., University of Chicago  
<http://astro.uchicago.edu/home/web/lucia/a100/lectures/copernicus.html>

“During the years between Ptolemy and Copernicus, many small epicycles had been added to the main epicycles to make Ptolemy's model agree with the observations.”  
Nick Strobel, Bakersfield College, Physical Science Dept.  
<http://www.astronomynotes.com/history/s4.htm>

“In order to describe these finer details, more than 80 epicycles were required.”  
Prof. Shane Stadler, Assistant Professor, Department of Physics, Southern Illinois University, Physics 101  
[http://www.physics.siu.edu/sstadler/PHYS\\_SP07/Lectures/Lecture1ca\\_nq.ppt](http://www.physics.siu.edu/sstadler/PHYS_SP07/Lectures/Lecture1ca_nq.ppt)

“By varying the size of epicycle and deferent, and speed of revolution on epicycle and deferent, Ptolemy was able to give an approximate representation of the planetary motion – requiring 80 circles! But the accuracy was not good! (failed to predict dates of eclipses etc.) The system was very cumbersome with all those 80 arbitrary numbers needed to describe it. The price to pay for retaining circular motion centered on Earth was large.”  
Astronomy 101: Introduction to Astronomy,  
Dr. Kathleen Flint, Center from the National Science Foundation & Stony Brook Southampton  
<http://ms.cc.sunysb.edu/~kflint/ast101/PDFs/lecture3.pdf>

# Quotes

“Ptolemy introduced a model of the solar system with 80 circles”, Slide 48, Astronomy 105  
John F. Wallin, Associate Professor of Space and Computational Science  
Dept. of Computational & Data Sciences, Dept. of Physics & Astronomy  
<http://astro.gmu.edu/classes/a10594/notes/105/105.html>

“This ultimately required 80 circles and nested epicycles, which was extremely complex and unwieldy”  
ASTR 111: Introduction to Modern Astronomy I,  
Peter A. Becker, Professor of Astrophysical, Planetary, and Space Sciences, George Mason University.  
[http://www.scs.gmu.edu/~pbecker/lecture4\\_handouts.pdf](http://www.scs.gmu.edu/~pbecker/lecture4_handouts.pdf)

“80 Uniform circles”  
ASTRO 1030, Chuck Higgins, Ph.D., Dept. of Physics and Astronomy, Middle Tennessee State University  
[http://physics.mtsu.edu/~higgins/teaching/astr1030/Lectures/Ch1\\_History\\_h3.pdf](http://physics.mtsu.edu/~higgins/teaching/astr1030/Lectures/Ch1_History_h3.pdf)

“more than 80 circles needed.” Physics 315, The Mysterious Universe  
Prof. Thomas Lockhart, Department of Physics and Astronomy, University of Wisconsin-Eau Claire,  
[http://www.uwec.edu/physics/lockhart/phys315/powerpoints/ancient\\_models.pdf](http://www.uwec.edu/physics/lockhart/phys315/powerpoints/ancient_models.pdf)

“Ptolemy required a series of 80 circles.”  
Astro 346, Martin Karl Wilhelm Pohl, Department of Physics and Astronomy, Iowa State University  
<http://cherenkov.physics.iastate.edu/~mkpohl/teach/346-s6/ch2.pdf>

“Kepler thus deftly reduces the 80 circles of Ptolemy, the 48 of Copernicus, along with their epicycles, deferents, eccentrics, and equants to”  
Drury University, Springfield, MO  
<http://www.drury.edu/ess/philsoci/PineCh4.html>

“used 80 epicycles to describe 5 planets, sun, moon”, Science/Physics 122, Prof. Richard Brill, Physical Science, Honolulu Community College

“Ptolemaic system for its "inordinately complex" heaps of epicycles on epicycles, but declares that in some respects it was more accurate than the Copernican system.” Simon Singh, BIG BANG, HarperCollins Publishers, (New York Times Review, The Real Creation Science By OWEN GINGERICH, Published: February 20, 2005)

# Quotes

“48 epicycles, compared to 40 in the Ptolemaic geocentric system”

Astronomy 161: An Introduction to Solar System Astronomy

Prof. Richard Pogge, Professor of Astronomy

The Ohio State University, Department of Astronomy

<http://www.astronomy.ohio-state.edu/~pogge/Ast161/Unit3/copernicus.html>

“Although these epicycles were much smaller than those in the Ptolemaic model, they were essential to precise prediction and result in a model that uses 48 circles (deferents plus epicycles), only 10 less than Ptolemy required.”

“However, Copernicus also used circular orbits and had to resort to epicycles and deferents to explain retrograde motions. In fact,

Copernicus was forced to use more epicycles than Ptolemy, i.e. a more complicated system of circles on circles.”

Astronomy 121: The Formation and Evolution of the Solar System,

Dr. James Schombert, Department of Physics, University of Oregon

<http://abyss.uoregon.edu/~js/ast121/lectures/lec02.html>

“Perhaps no other subject in the history of science has had more distortions and misunderstandings than the development of ancient astronomy. For example, one of the most persistent canards is that Ptolemy's model of planetary motion required up to 80 epicycles which prompted the search for simpler models.”

“Gingerich attributed to Copernicus the "aesthetic" idea that "celestial motions should be described in terms of uniform circular motion", but as Evans pointed out this idea was the basis for celestial motions of Greek astronomy. “

Michael Nauenberg, University of California Santa Cruz

<http://physics.ucsc.edu/~michael/longbeach.html>