THE PROPAGATION OF GRAVITY IN SPACE AND TIME

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The Basic Law

The gravitational phenomena are the only examples of action-at-a-distance for which no changes have been detected in the space between the component bodies. It is, therefore, understandable that we are confident that such changes will someday be demonstrated. Thus we should not view the subject as if gravitation is an exceptional case: just because such changes are not now demonstrable, that does not mean that we should doubt their existence. All known and understood evidence compels us to believe the opposite. However much that conclusion may rest on missing experience and incomplete analysis, it is first of all necessary to demonstrate that there are facts which justify and complete the above view, that is, that gravitational phenomena are observable. Thus it is necessary to avoid all hypotheses which go beyond the supposition that the space between two gravitating masses participates in the gravitational action. An earlier but inadequate treatment of this question can be found in an article entitled "Fernwirkungen," by Drude, in the proceedings of the 69th *Naturforscherversammlung*.

Two gravitating masses are recognized by their resistance to an increase of their separation. Thus, though the masses themselves may either be at rest or moving, whatever occurs in the space between the bodies must be continuous. Clearly, at any given position, or position plus equations of motion, not only is the localized resistance determined at that position, but the value of said resistance can be determined anywhere between the two bodies. The amount of work necessary to overcome this resistance is that amount characteristic of the gravitational field. Since we are concerned with time dependent gravitational changes in space, this work can be regarded as a fundamental quantity. Furthermore, in this conceptual framework it makes no sense to speak of the resistance of space, since such resistance (inertia) is only present at the positions of the masses. If one speaks of an event requiring time to proceed from points 1 and 2, then it is equivalent to saying that the event ceases to exist at point 1 before it reaches point 2. Hence the energy stored in the event would have to disappear if it did not pass through the space between points 1 and 2. This energy is equal in magnitude to the aforementioned amount of work if the event is related to the gravitational effect of the two masses at their respective positions in space. This is because the amount of work is a function of the separation of the bodies as well as their motion; and this work cannot demand to different values for the energy.

For the sake of discussion, we shall call the first mass the attracting one and the other we shall refer to as the attracted mass. By the potential V of the attracting mass, m, we mean the amount of work needed to separate the two masses out to infinite separation. The total potential is then V_m. Assuming mass m to be at rest, we shall assign it a position x, y, z in the coordinate system centered on the attracting mass with that mass assumed to be at rest.

Using Mach's principle as outlined in his "Prinzipien der Wärmelehre" (Principles of Heat Theory) we can calculate V. In particular, we want it to be the average value of all the potentials in some small neighborhood about a given point. V is not a vector and for the above situation, it is also not a function of time. At the mass, m, it has a value of f(x, y, z) while at some nearby point it has a value of f(x+h, y+k, z+*l*). Furthermore,

$$\psi\left(\sqrt{h^2+k^2+l^2}\right)$$

can be defined as the average weight of this point which, because of the small distances involved, rapidly diminishes as the distance increases.

Thus we find:

$$V = \frac{\iiint_{-\infty}^{+\infty} f(x+h, y+k, z+l)\,\psi(\sqrt{h^2+k^2+l^2})\, dhdkdl}{\iiint_{-\infty}^{+\infty} \psi(\sqrt{h^2+k^2+l^2})\, dhdkdl} .$$

If we develop f to second order via a Taylor series, and then integrate about the point x, y, z, we obtain:

$$\iiint_{-\infty}^{+\infty} \left(\frac{df}{dx}h + \frac{df}{dy}k + \frac{df}{dz}l\right) \psi(\sqrt{h^2+k^2+l^2})\, dhdkdl = 0,$$

$$\iiint_{-\infty}^{+\infty} \frac{df}{dx}\frac{df}{dy}hk + \frac{df}{dx}\frac{df}{dz}hl + \frac{df}{dy}\frac{df}{dz}kl\psi(\sqrt{h^2+k^2+l^2})\, dhdkdl = 0,$$

$$\iiint_{-\infty}^{+\infty} \psi(\sqrt{h^2+k^2+l^2})\, h^2 dhdkdl = \iiint_{-\infty}^{+\infty} \psi(\sqrt{h^2+k^2+l^2})\, k^2 dhdkdl =$$

$$\iiint_{-\infty}^{+\infty} \psi(\sqrt{h^2+k^2+l^2})\, l^2 dhdkdl .$$

If one takes:

$$\frac{\iiint_{-\infty}^{+\infty} \psi(\sqrt{h^2+k^2+l^2})\, h^2 dhdkdl}{\iiint_{-\infty}^{+\infty} \psi(\sqrt{h^2+k^2+l^2})\, dhdkdl} = n$$

then there remains:

$$V = V + \frac{n}{2}\left(\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2}\right),$$

and so,

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0.$$

From this equation it follows that if μ is a constant and r is the distance between the masses, that:

$$V = \frac{\mu}{r}.$$

This leads us to Newton's law of gravitation, for $V=\mu/r$ is still in effect at the moment at which the masses are released. The increase of V_m coincides with the appearance of a force dT and that, therefore, neither T nor V contain the time-dependence at the moment of release. In accordance with Lagrange's general equations of motion, when we replace the external force on mass m by the negative value of the force exerted by it, we obtain the following equations for the acceleration of mass m:

$$\frac{1}{m}\frac{dT}{dr} = \frac{dV}{dr} = -\frac{\mu}{r^2} .$$

Newton's law will yield the potential which the masses will achieve in their available time. This condition is always fulfilled when the masses, with a fixed distance between them, are held at rest. This ceases to be the case if the bodies recede from each other freely, and if the aforementioned time is finite and measurable. Two circumstances then influence the situation. In the first place, at a distance r-Δr (Δr being positive for the receding case and negative for the approaching case) the new potential must still be inversely proportional to the new distance, r-Δr. If this were not so, then it would be impossible to understand how the stationary case could work. But this potential does not immediately exert a force on m, because the event which caused the change in the potential needs some time to travel from the attracting mass to the attracted mass. It goes without saying that this is also the case if the potential proceeded from the attracted to the attracting mass. In the same way heat radiated from one body to another requires some sort of recoil or counterradiation. The potential emanating fro the attracting mass over the distance r-Δr manifests itself at mass m only at some time interval, Δt, later at which time the distance has become equal to r.

In the second place, it is possible that in the case of action-at-a-distance the potential would immediately appear to its full extent, but because space and time do play a role in the assumed action, surely the potential needs some time interval in order to effect the mass, m; that is, to allow m to manifest its responsive motion. Only the assumption of action-at-a-distance allows for simultaneity. The most important consequence of introducing the concept of local action as a replacement for action-at-a-distance is that it introduces, for gravity, the same sort of event propagation that is evidenced in other physical and chemical processes. In the case of a thrust, the thrusting force then consists of a succession of elementary thrusts; in like manner, the change in potential communicates itself to m as a rapid succession of potential differentials.

In the case where the masses are at rest, the potential passes m with its own characteristic velocity and so the potential communicated to m must vary inversely with distance. If the masses move towards each other, then the travel time to traverse their separation diminishes and so the potential which is felt at m decreases by the ration of the potential's velocity to said velocity plus the relative velocity of the two masses.

Apart from its own velocity the potential also moves with the velocity of the attracting mass from which it emanates. The path, r-Δr, covered by the opposing motions of the potential and the attracting mass during a time interval, Δt, amounts to:

$$\Delta t\left(c - \frac{\Delta r}{\Delta t}\right);$$

where r=cΔt. Thus one obtains for the distance over which the potential is changing and the distance over which it is still inversely proportional to distance:

$$r - \Delta r = r\left(1 - \frac{1}{c}\frac{\Delta r}{\Delta t}\right) .$$

Furthermore, the velocity with which the motions are communicated to each other is:

$$c - \frac{\Delta r}{\Delta t}$$

and therefore the potential, because it takes time to communicate itself to m, turns out to be the following ratio:

$$\frac{c}{c-\dfrac{\Delta r}{\Delta t}}$$

Thus we arrive at:

$$V = \frac{\mu}{r\left(1 - \dfrac{1}{c}\dfrac{\Delta r}{\Delta k}\right)^2}$$

As long as the distance, Δr, is small, so that Δr/Δt is small relative to c (the speed of propagation), the differential, dr/dt, can be used. As a result, V becomes:

$$V = \frac{\mu}{r\left(1 - \dfrac{1}{c}\dfrac{dr}{dt}\right)^2} ,$$

which, when expanded to second order, becomes:

$$V = \frac{\mu}{r}\left[1 + \frac{2}{c}\frac{dr}{dt} + \frac{3}{c^2}\left(\frac{dr}{dt}\right)^2\right] .$$

Here the expression for V is not only a function of r, but also a function of the time derivative of r. From Lagrange's equation we can solve for the acceleration of m so that, writing r' for dr/dt, we obtain:

$$\frac{1}{m}\frac{dt}{dr} - \frac{1}{m}\frac{d}{dt}\frac{dT}{dr'} = \frac{dV}{dr} - \frac{d}{dt}\frac{dV}{dr'} = -\frac{\mu}{r^2}\left[1 - \frac{3}{c^2}\left(\frac{dr}{dt}\right)2 + \frac{6r}{c^2}\frac{d^2r}{dt^2}\right].$$

The assumption that dr/dt << c is valid for normal gravitational phenomena. If it were not so, then Newton's law of gravitation would not work the way it does. Under certain conditions, however: for instance if the initial velocity imparted to the masses comes from outside of the system, then dr/dt may become so great that we are not allowed to equate dr/dt with $\Delta r/\Delta t$, nor could we restrict the expansion to second order. The above formula is thus only valid as long as the masses form an isolated system, not influenced by external effects. In the important case at hand, this formula describes the change which Newton's law undergoes as a result of the fact that the potentials between the masses are not propagated instantaneously but require an interval of time.

The Speed of Propagation

Depending upon whether measurements of c are finite or infinite (as c is defined in the above equations,) we shall conclude that the potentials either need time to traverse the distance between the masses or that no such time is needed. If the latter should prove to be the case, then gravity is an example of action at a distance. Two conditions must be fulfilled. First, because c>>dr/dt, we need to separate the terms for the acceleration of the mass m from the total expression and to make them compatible with observations. In the second place, we need to estimate to some order of magnitude the possible finite value of c. After that, we need to test it. So far, the only examples we have are the planets, and thus we shall assume the sun to be the attracting mass and we shall take on e of the planets to represent the attracted mass. To simplify the matter, we shall take the sun to be located at the origin of the coordinate system so that the constant, μ, has a greater value; containing the greater mass.

One writes:

$$\frac{3}{c^2}\left(\frac{dr}{dt}\right)^2 - \frac{6r}{c^2}\frac{d^2r}{dt^2} = F.$$

Therefore:

$$\frac{d^2x}{dt^2} = -\frac{\mu x}{r^3}(1 - F),$$

$$\frac{d^2y}{dt^2} = -\frac{\mu y}{r^3}(1 - F),$$

Multiplying the one equation by y and the other by x, land subtracting, it follows that:

$$x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = 0.$$

This is also the equation which is derived for planetary orbits by Newton's law. By integration and converting to polar coordinates where θ is the angle between the radius vector and the abscissa and L is a constant, we derive:

$$r^2\frac{d\theta}{dt} = L.$$

This can be written in the form:

$$dt = \frac{r^2 d\theta}{L}$$

and furthermore, with

$$\frac{x}{r} = \cos\theta \quad \text{and} \quad \frac{y}{r} = \sin\theta$$

being substituted in the equations for

$$\frac{d^2x}{dt^2} \quad \text{and} \quad \frac{d^2y}{dt^2}$$

these become:

$$d\frac{dx}{dt} = -\frac{\mu}{L}(1 - F)\cos\theta d\theta ,$$

$$d\frac{dy}{dt} = -\frac{\mu}{L}(1 - F)\sin\theta d\theta .$$

Integration yields:

$$\frac{dx}{dt} = -\frac{\mu}{L}\sin\theta + \left(M + \int \frac{\mu}{L} F\cos\theta d\theta\right),$$

$$\frac{dy}{dt} = +\frac{\mu}{L}\cos\theta + \left(N + \int \frac{\mu}{L} F\sin\theta d\theta\right).$$

where M and N are constants of integration. Since L = x(dx/dt) - y(dy/dt), the latter two equations yield:

$$r = \frac{L}{\frac{\mu}{L} - \left(M + \int \frac{\mu}{L} F\cos\theta d\theta\right)\sin\theta + \left(N + \int \frac{\mu}{L} F\sin\theta d\theta\right)\cos\theta} .$$

If F does not vanish, then the integrals in the denominator take on progressively differing values. If we assume that their values are known at some given time, then we can take it that the planet can be found somewhere on the ellipse described by this equation and at that given time. Let a be the semi-major axis and let b be the semi-minor axis: also let ε be the eccentricity and take ω as the angle between a and the abscissa; then the equation can be solved for:

$$r = a(1-\varepsilon), \quad r = a(1+\varepsilon)$$

and $r=b^2/a$ in accordance to:

$$L, M + \int \frac{\mu}{L} F\cos\theta d\theta$$

and

$$N + \int \frac{\mu}{L} F\sin\theta d\theta$$

so we obtain:

$$L = b\sqrt{\frac{\mu}{a}} ,$$

$$M + \int \frac{\mu}{L} F\cos\theta d\theta = -\frac{\epsilon}{b}\sqrt{a\mu}\sin\omega ,$$

$$N + \int \frac{\mu}{L} F\sin\theta d\theta = \frac{\epsilon}{b}\sqrt{a\mu}\cos\omega .$$

It will be seen when we consider the constancy of b/a^2, that we can explain the motion of the planet as if it were moving on an ellipse for which ε and ω are changing continuously. Only if F=0 does this change stop. Therefore, it is this change by which the finite value of c becomes demonstrable. Differentiating the last two equations conformably to θ, substituting in the value for L and dividing the one expression by

$$\cos\theta = \sqrt{\frac{a\mu}{b}} ,$$

and the other by

$$\sin\theta = \sqrt{\frac{a\mu}{b}} ,$$

and solving for F yields:

$$F=\frac{\sin\omega}{\cos\theta}\frac{d\epsilon}{dt}\frac{dt}{d\theta}-\frac{\epsilon\cos\omega}{\cos\theta}\frac{d\omega}{dt}\frac{dt}{d\theta},$$

$$F=\frac{\cos\omega}{\sin\theta}\frac{d\epsilon}{dt}\frac{dt}{d\theta}-\frac{\epsilon\sin\omega}{\sin\theta}\frac{d\omega}{dt}\frac{dt}{d\theta},$$

Setting $\alpha = \theta - \omega$ and combining the expressions yields:

$$\frac{d\epsilon}{dt} = -\epsilon\tan\alpha\,\frac{d\omega}{dt}$$

which, when substituted into the former equations, yields:

$$F=-\frac{\epsilon}{\cos\alpha}\frac{dt}{d\theta}\frac{d\omega}{dt}$$

In order to use this to obtain a relationship for $d\omega/dt$ which involves only observable parameters one needs to express F in terms of the derivative of r with respect to t. Since b/a^2 is constant, and using the relationships:

$$\frac{d\epsilon}{dt}=-\epsilon\tan\alpha\frac{d\omega}{dt},$$

$$r^2\frac{d\theta}{dt}=L \quad \text{and} \quad L=b\frac{\sqrt{\mu}}{\sqrt{a}}:$$

$$r=\frac{\frac{b^2}{a}}{1+\epsilon\cos\alpha},$$

$$\frac{dr}{dt}=-\frac{ar^2}{b^2}\left(\cos\alpha\frac{d\epsilon}{dt}-\epsilon\sin\alpha\frac{d\theta}{dt}+\epsilon\sin\alpha\frac{d\omega}{dt}\right)$$

$$=-\frac{ar^2}{b^2}\left(-\epsilon\cos\alpha\tan\alpha\frac{d\omega}{dt}-\epsilon\sin\alpha\frac{d\theta}{dt}+\epsilon\sin\alpha\frac{d\omega}{dt}\right)$$

$$=\frac{a\epsilon r^2}{b^2}\sin\alpha\frac{d\theta}{dt}$$

$$=\frac{\epsilon\sqrt{a\mu}}{b}\sin\alpha,$$

$$\frac{d^2r}{dt^2}=\frac{\sqrt{a\mu}}{b}\sin\alpha\frac{d\epsilon}{dt}+\frac{\epsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\theta}{dt}-\frac{\epsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\omega}{dt}$$

$$=-\frac{\epsilon\sqrt{a\mu}}{b}\sin\alpha\tan\alpha\frac{d\omega}{dt}+\frac{\epsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\theta}{dt}-\frac{\epsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\omega}{dt}$$

$$=-\frac{\epsilon\sqrt{a\mu}}{b}\sin\alpha\tan\alpha\frac{d\omega}{dt}+\frac{\epsilon\mu}{r^2}\cos\alpha-\frac{\epsilon\sqrt{a\mu}}{b}\cos\alpha\frac{d\omega}{dt}$$

$$= -\frac{\epsilon\sqrt{a\mu}}{b\cos\alpha}\frac{d\omega}{dt} + \frac{\epsilon\mu}{r^2}\cos\alpha \,.$$

Therefore:

$$F = \frac{3\epsilon^2 a\mu}{b^2 c^2}\sin^2\alpha + \frac{6\epsilon r\sqrt{\alpha\mu}}{bc^2}\cos\alpha\frac{d\omega}{dt} - \frac{6\epsilon\mu}{rc^2}\cos\alpha \,.$$

Hence the desired expression for dω/dt is:

$$\frac{\epsilon r^2\sqrt{a}}{b\sqrt{\mu}\cos\alpha}\frac{d\omega}{dt} = -\frac{3\epsilon^2 a\mu}{b^2c^2}\sin^2\alpha - \frac{6\epsilon r}{bc^2}\frac{\sqrt{a\mu}}{\cos\alpha}\frac{d\omega}{dt} + \frac{6\epsilon\mu}{rc^2}\cos\alpha$$

or, after substituting in $r = \dfrac{b^2}{a(1+\epsilon\cos\alpha)}$ and $b = a\sqrt{1-\epsilon^2}$

and after dividing by: $\dfrac{\epsilon r^2\sqrt{a}}{b\sqrt{\mu}\cos\alpha}$

$$\frac{d\omega}{dt} = -\frac{6\mu}{a(1-\epsilon^2)c^2}(1+\epsilon\cos x)\frac{d\omega}{dt} - \frac{3\epsilon\mu^{3/2}\cos\alpha}{a^{5/2}(1-\epsilon^2)c^2}(1+\epsilon\cos\alpha)^2\sin^2\alpha$$

$$+ \frac{6\mu^{3/2}}{a^{5/2}(1-\epsilon^2)^{5/2}c^2(1+\epsilon\cos\alpha)^3\cos^2\alpha} \,.$$

In order to compare the calculated angular speed dω/dt with observation, recall that the above situation involves only one planet. So it follows that only those effects that cause perihelion precession, but are not due to perturbations, can be considered. Only for Mercury is this quantity known, and that value for the perihelion precession is 41 seconds of arc per century.. Such a small angle precludes direct observation of the slow but steady change in dω/dt. Hence a longer integration time is needed. In the last equation, ε appeared but not dε/dt and insofar as ε>>dε/dt, ε can be taken as constant. Thus it is sufficient to select the limits of integration as α=0 and α=2π because dω/dt almost repeats itself with every subsequent revolution.

Multiplying dω/dt by dt, and using

$$dt = \frac{r^2}{L}d\theta = \frac{a^{3/2}(1-\epsilon^2)^{3/2}}{\mu^{1/2}(1+\epsilon\cos\alpha)^2(d\alpha + d\omega)} \,,$$

in the second and third terms, and by properly ordering the terms and dividing, one arrives at:

$$d\omega = \frac{\frac{6\mu}{a(1-\epsilon^2)c^2}(1+\epsilon\cos\alpha)\cos^2\alpha - \frac{3\epsilon\mu}{a(1-\epsilon^2)c^2}\sin^2\alpha\cos\alpha}{1+\frac{6\mu}{a(1-\epsilon^2)c^2}(1+\epsilon\cos\alpha) - \frac{6\mu}{a(1-\epsilon^2)c^2}(1+\epsilon\cos\alpha)\cos^2\alpha + \frac{3\epsilon\mu}{a(1-\epsilon^2)c^2}\sin^2\alpha\cos\alpha}$$

Dividing the numerator and denominator through by:

$$\frac{3\mu}{a(1-\epsilon^2)c^2} = \frac{\gamma}{c^2},$$

and ordering the terms with respect to increasing powers of $\cos\alpha$ and letting:

$$v = -\epsilon\cos\alpha + 2\cos^2\alpha + 3\epsilon\cos^3\alpha$$

$$w = 3\epsilon\cos\alpha - 2\cos^2\alpha - 3\epsilon\cos^2\alpha$$

then

$$d\omega = \left[\frac{v}{\frac{c^2}{\gamma}+2+w}\right]d\alpha .$$

This is approximately equal to:

$$d\omega = \left[\frac{v}{\frac{c^2}{\gamma}+2} - \frac{vw}{\left(\frac{c^2}{\gamma}+2\right)^2}\right]d\alpha .$$

Thus the perihelion precession value for one revolution is:

$$\psi = \int_0^{2\pi}\left[\frac{v}{\frac{c^2}{\gamma}+2} - \frac{vw}{\left(\frac{c^2}{\gamma}+2\right)^2}\right]d\alpha .$$

or, because

$$vw = -3\epsilon^2\cos^2\alpha + 8\epsilon\cos^3\alpha + 4(3\epsilon^2-1)\cos^4\alpha - 12\epsilon\cos^5\alpha - 9\epsilon^2\cos^6\alpha ,$$

$$\psi = \frac{2\pi}{\frac{c^2}{\gamma}+2} + \frac{3\pi(8-\epsilon^2)}{8\left(\frac{c^2}{\gamma}+2\right)^2} .$$

Since ψ is small, the second order term lunder the radical is very much less than the first so that it can be ignored. The approximate expression derived for $d\omega$ is, therefore, still too precise. That is, we could have ignored it from the outset. Consequently,

$$c^2 = \frac{2\pi\gamma}{\psi} - 2\gamma ,$$

$$\frac{c^2}{\gamma} + 2 = \frac{2\pi}{\psi},$$

in which case the 2γ is negligible relative to the $2\pi\gamma/\psi$ term. Finally,

$$c^2 = \frac{6\pi\mu}{a(1-\epsilon^2)\psi}$$

wherein

$$\mu = \frac{4\pi^2 a^3}{\tau^2},$$

if τ is taken to be the period of the planet. For Mercury we have:

$$a = 0.3871 \times 149 \times 10^6 \text{ km},$$
$$\varepsilon = 0.2056$$
$$\tau = 88 \text{ days}$$
$$\Psi = 4.789 \times 10^{-7}.$$

This yields:

$$c = 305{,}500 \text{ km/sec}.$$

Foucault's value for the velocity of light of 298,000 km/sec is the smallest value found to date; the largest is found by Römer's method and has a value of 308,000 km/sec. According to Hertz, the velocity of electromagnetic waves is 320,000 km/sec. *Thus the velocity with which a gravitational potential is propagated agrees with the velocity of light as well as with the velocity of electric waves.* At the same time, this result guarantees that such a velocity does exist. Needless to say, no one can deny that the perihelion shift of Mercury, 41" per century, is still possibly the result of as yet unknown circumstances, and that as such the value of c still could be infinite when dealing with gravitational potentials; but it should be kept in mind that the formula for the dependence of that potential on the calculated speed has not been derived through some complicated hypothesis but, instead, came about in a natural way; however the result may differ from those of previous investigators. It would be an unusual coincidence if the 41" of Mercury should exactly deliver the same velocity as that of light and electricity without there being any spatial and temporal propagation of gravity; for the medium in which this propagation occurs is the same space between the celestial bodies in which the propagation of light and electricity happen. Even the relatively large perihelion precession which is required by the above analysis for the planet Venus, namely 8"/century, cannot be used as much of an objection unless a revision of the analysis of the perturbations of that planet should indicate that such a large value is definitely precluded.[48] In this context the reader is reminded that the secular acceleration of the moon varies from 6" to 12". For the rest of the planets the perihelion precessions are imperceptibly small. They can be readily obtained from the standard tables and are the following values per century: for the earth, 3".6, for the moon 0".06, for Mars 1".3, for Jupiter 0".06, for Saturn 0".002 and for Neptune 0".0007.