

TWO TREATISES ON MECHANICS¹

Published by *Pierre Prevost*,
As publisher of the first, and as author of the second treatise

FIRST TREATISE — MECHANICS

by *George-Louis Le Sage*

Drafted from his notes and containing

- 1 . The description of his system of ultramundane corpuscles
2. The refutation of objections that could be made to this system
3. The theory of elastic fluids, according to the same system.
4. Some applications of the same system to certain affinities.

NOTE BY THE PUBLISHER

- 1 . To the subjects listed at the head of this Mechanics, I had hoped to add a study by Le Sage *On Cohesion*, in which he sets out to prove that this great phenomenon cannot be explained by Newtonian attraction; but the drafting of this study could not be completed in time.
2. The Public Library of Geneva has agreed to accept my offer of the deposit of the manuscript of G. L. Le Sage. This deposit will be made shortly. Two advantages will result. The first, that persons interested in the published works of this author will be able to have recourse to the deposit to obtain more details. The second, that the rest of the manuscripts of this prolific scientist will not perish, and will, perhaps, find a publisher after me.

Book I

Chapter One

DESCRIPTION OF THE SYSTEM OF ULTRAMUNDANE CORPUSCULES

Article 1

Object of the system

1. The object of this system is to explain the phenomenon known by the name of *Newtonian attraction*. This phenomenon, which could be more simply called by the general word, *gravity*, includes *weight* as a particular case.

2. The general laws of Newtonian attraction are the law of masses, the law of distances and the law of acceleration.

The first is stated in the following terms: *In all material objects observed, the force that makes them tend to approach each other is proportional to their masses.*

¹ Translated from French by Mr. M. Marling with technical refinements by Prof. James N. Hanson and Dr. Gerardus D. Bouw. First published between 1985 and 1987 in *The Bulletin of the Tychonian Society*. Some corrections and explanatory footnotes have been added for this A. D. 2005 edition.

The second is as follows: *The force by which a body is impelled towards another body is inversely proportional to the square of their separation.*

The law of acceleration, first observed in bodies gravitating towards the earth, then assumed and verified in the heavens, is the following: *The distance covered by a body during its free fall varies as the square of the length of time it has been falling.* And it is shown that the equality of the elemental impulses with weight is the result of this law, and also, reciprocally, that the law results from the equality of these elemental impulses.

Such are the laws, or the expressions of the general relations to which all the phenomena of gravitation are related. If, therefore, it is desired to explain these phenomena, it is pointless to go into minor details.

Consideration should be limited to the application of the proposed cause to the laws just enunciated, and thereafter all the multitude of dependent phenomena will be fully explained.

Article 2

Constitution of the gravific fluid

3. Space, being conceived empty, it is necessary first of all to place therein a very small *atom*, of the kind physicists usually call *hard* in the absolute sense, that is to say, unbreakable, inflexible and having no elasticity.

The shape of this body is not determinable by any phenomenon. It can, for the sake of simplicity, be conceived to be spherical. Picture this first atom, or *corpuscule*, being thus constituted, being arranged in uniform and regular order with other similar corpuscles in such a way as to fill the whole of space, leaving, however, large intervals between each atom and the atoms next to it. You now have the concept of a discrete fluid, with its particles at rest.

Impart to each corpuscule an equal impulse, thus communicating to each of them the same very great speed (great relative to all known or observed speeds). Imagine that the velocities thus imparted are randomly oriented so that there is no gradient to the resulting flow. The discrete fluid constituted in this manner, with each of its components moving at equal and very high speed, crosses the universe; consequently, it has come from places beyond the world and that is why these are termed *ultramundane corpuscles*. The fluid, itself, is called *gravific* because it produces gravity.

4. It is necessary to pause here for a moment to consider the general effect of the constitution we have just described. Consider an arbitrary point in the universe. However short the moment of our contemplation, because of the rapidity of the ultramundane corpuscles, we can say that through that point and for that moment there pass streams of corpuscles from every conceivable direction. Although this may not be strictly true, it is true enough for all practical purposes.

Thus we can say that at any arbitrary point, at any time, that point perceives itself as a center on which converge and from which diverge countless corpuscles: that is to say, that there arrive at that point and at that instant, a multitude of corpuscles from all directions in space; and that likewise a multitude of corpuscles leaves it in all directions so that the net flux is zero.

Article 3

Explanation of the general phenomenon

5. Having conceived the gravific fluid constituted in this way, let us plunge into this fluid a solid body with no corners or sharp angular aspects and/or bounded by convex surfaces (such as a

sphere, for example). Let the body be much larger than the corpuscular size. This body will remain motionless, or at least, not subject to any drift. It will perhaps be tossed about by irregularities in the currents and may thus exhibit irregular oscillations.

Plunge a second, similar body into the fluid at some distance from the first. These two bodies will approach each other; for the one shields the other, and those currents no longer having counter-acting currents become necessarily more effective, producing in both bodies a constant movement whereby they tend toward each other.

Article 4

Explanation of the law of distances

6. Let us now examine more closely the consequences of the existence of the corpuscles. Each point in the physical world clearly occupies the center of this immense sphere of ultramundane corpuscles so that the flux is isotropic.

Assuming that there is one particle of matter, much too small for our senses to perceive but much bigger, however, than an ultramundane corpuscule; and assuming that this particle occupies a given point in space; it will stop all the corpuscles advancing towards the point. In effect, they can all be regarded as having been intercepted. Those going towards the particle can be conceived of as though they were successively passing through different spherical surfaces having the particle as a common center: and the corpuscles passing through anyone of these surfaces are exactly the same as those that have passed through all the other, more radially-distant surfaces. From this perspective, the flux can be seen to concentrate as one gets nearer to the point and considers successively smaller spheres. The surface areas of the spheres are related to the square of their radii which, in our model, are the distances of the spherical surfaces from our particle.

Therefore, the flux of the ultramundane corpuscles at different distances from the particle vary inversely to the square of the distances. Thus, finally, their momentum carried to the particle and encountering an intervening body will be inversely proportional to the square of the distance of that body from the particle.

7. This last consequence is the sole proposition from which Newton deduced the Laws of Kepler, all the recurrent celestial phenomena, and the tides.² The laws governing the fall of sublunary³ bodies are only a minor corollary of this proposition.⁴ It would be pointless to repeat here the consequences of this great principle, as I could not differ from the many authors who have deduced them except in some of my expressions. I would, for example, give the names *impermeability*, *interception*, *impulsion*, and *approach* to what they term *attractive force*, *stress*, and *gravitation*. And I would speak of *impermeable particles* and *polygons* instead of *points of attraction* and *curves*.

The consistency of the consequences deduced from this theory would be enough to satisfy not only the apparent consistency due to the imperfections of our senses, but satisfying an even greater consistency, if that should prove necessary, which is to say, if we had some certainty that such a

² As regards phenomena that can continue by virtue of their inertia alone, without the need of any new impulse, these must be related directly to a motion given them once and for all by the first cause. Such are the size and density of the planets, their distance from the sun, the direction and eccentricity of their orbits, the direction and speed of their rotation, etc.

³ I.e., moving less than the speed of the corpuscles or light.

⁴ These laws basically reduce to the direction of heavy bodies, as determined by the Law of distances (Newton, *Princ., Lib. I, prop. 71*), and the theorems of Galileo which, when the distances are approximately equal, is the consequence of the equality of the shocks of the gravific fluid (see Article 7).

greater consistency is, in fact, necessary. There could be as much precision, for example, as encountered in optics, where we reason along physical lines as if they were mathematical.

Having thus explained, via ultramundane corpuscles, the Law of Newtonian attraction insofar as distance is concerned, we now pass on to a consideration of the law as relating to masses. But this calls for us to digress and consider the constitution of massive bodies.

Article 5

Concerning the constitution of massive bodies

8. Several factors testify to the great porosity of material objects. The pores must, moreover, be conceived to be so constructed as to allow the ultramundane corpuscles easy passage. Hence they should be very permeable to these corpuscles.

The elements should, therefore, be conceived to be so ordered as to leave interstices that are very large relative to the diameter of the corpuscles.

Furthermore, the elements should also be conceived of as very permeable themselves, akin to a cage composed of bars, having a very small cross-section in relation to the distance between the constituents of the elements.

From these concepts it will be noted that the great bodies (such as the terrestrial globe) will only stop or intercept a very small part of the flux of corpuscles which pass through them.

Article 6

Explanation of the Law of masses

9. It follows from the constitution of massive bodies that the number of corpuscles that arrive at the first and last layers of a body is substantially the same, regardless of the thickness of the body. Consequently, the collisions are proportional to the quantity of matter; in other words, the weight thus evidenced is proportional to the mass.

10. For the rest, I agree that, according to this system, this law ought not to be rigorous; but there is nothing to prove that it is rigorous. The most exacting observations have only proven it to be so to $1/1,000^{\text{th}}$ in the vicinity of the earth, and perhaps $1/10,000^{\text{th}}$ in celestial bodies. It can be said that atoms no doubt pass very freely through all massive bodies, as freely, for example, as light passes through a diamond, and magnetic matter through gold; although one of these materials is the hardest and the other the densest of all known bodies (which proves that they have fewer pores than most of the latter); so that the number of corpuscles intercepted by the first layer of a massive body is absolutely imperceptible in proportion to the number of corpuscles that reaches the last layer.⁵ However, the particles intercepted by the first layer do have the force to produce a noticeable action on the first layer because of the momentum imparted on them by their immense speed, albeit not from their tiny mass.

Article 7

Acceleration of massive bodies

11. The Law of acceleration of massive bodies is also explained in this system with all necessary precision. It must be remembered that the ultramundane corpuscles all have the same speed, that this speed is very great in relation to all known and observed speeds, and that each

⁵ Newton, *Princ.*, Lib. III, prop. 6.

corpuscule is extremely small relative to the smallest particles constituting material bodies, and still more so in relation to the small aggregates of these particles.

This being so, when a corpuscule impacts a mass-particle forming part of a body, it can only give it an extremely low speed (very approximately, the reciprocal of their masses). Thus, the second corpuscule of the same stream, reaching the same particle, will give it the same shock as the first; and it will be the same with those that follow. Thus, the successive shocks are all equal. This is enough, however, to fully explain the law of acceleration.

12. It is true that this law in this system is not exact; but it can approach being so as closely as is necessary for any observed phenomenon. All that is needed is enough speed for the corpuscules and that they be small enough; two conditions that can only be determined by the phenomena themselves.

The following chapters provide a more detailed explanation of the chief points of this same system.

Chapter Two

Qualitative derivations of my Corpuscular Physics

Article 1

General view of the smallness of the ultramundane Corpuscules, their speed, and the rarity of the Gravific Fluid

13. From the law of masses (the law by which weight is proportional to the amount of matter), it follows that the particles of the gravific fluid are extremely small; for only through great permeability can the gravific corpuscules penetrate into the interior of masses to act on the shielded interior particles with a force clearly equal to that with which they press surface particles. The permeable pores of dense materials (such as gold) and even of less dense materials (such as pumice, cork, etc.) are, however, very small in proportion to what can be perceived by our senses. Thus the ultramundane corpuscules must be singularly small since they are rarely captured when passing through not just these miniscule pores or traversing a few, thin, surface layers, but able to pass clear through the terrestrial globe, and even through the globe of the sun.

The immediate conclusion drawn from this is that the corpuscules must either be very dense or else moving very fast or are both fast and dense. The latter is the most likely, for weight and Newtonian attraction, which is a kind of weight, are powerful forces. If, then, they are the result of impacts of very small corpuscules, then the impacts must at least be powerful in other ways: in this respect we consider that the density, without high speed, will probably not suffice to explain the observations. Density has its limits. The most that can be done is to attribute to each corpuscule mass without any empty space inside it. The fundamental particles of bodies, which the corpuscules can hit, are also necessarily to a great part solid or massive. They must be assumed, however, to be much larger than the corpuscules if these latter are to have the ability to easily traverse the pores in the fundamental particles. Therefore, the impact of a single corpuscule on such a particle would be very weak if the corpuscule were not also traveling at high speed.

14. Once a high speed is assumed, there is no difficulty in assuming a low density in the gravific fluid. Indeed, the phenomena do not require real continuity, but only apparent or

perceptible continuity of gravity. Thus, the corpuscles that follow each other along the same path—those comprising the same stream—can be separated from each other by large distances as long as those distances are traversed virtually instantaneously. This low density, thus theoretically possible, is necessary to explain the perfect freedom with which movement takes place—particularly the movement of the corpuscles themselves. All the arguments put forth by those who argue the low density and discreteness of the luminous fluid can be applied here with added force. The streams of corpuscles must be taken to cross each other in all directions without shielding each other because weight and attraction act in all directions in space. Besides, the ease of movement of all other bodies requires space to be an almost perfect void.

15. It can thus be said that the system of ultramundane corpuscles affords the most perfect vacuum in the universe consistent with observed phenomena and, at the same time, a type of plenum produced by the speed of the corpuscles which plenum is such that all tangible points of the created universe are always occupied. Indeed, if the system is properly conceived, it should be noted that no point is ever empty (provided it is a question of perceptible moments and points and not strictly a null part). This “fullness” is no doubt the only one that makes the phenomena possible.

Article 2

Some more Precise Parameters

16. Before considering the parameters, we must note that these exhibit only one type of limit and that no phenomenon affords us two limits. We can show that the size of the ultramundane corpuscles is smaller than some given value; that their speed exceeds a certain value; that the density of the gravific fluid exceeds another value; but we cannot say that the corpuscles are larger, slower, or less dense than some set of values.

17. The following reveals the speed-limit of the corpuscles: Consider two pendulums, one four times the length of another but each describing similar arcs; then not only is the absolute speed of one twice that of the other, but the vertical component of the speed of the longer will also be twice that of the shorter. Thus, when the pendulum bobs are in the descending portion of their swings, they reduce the impact of the corpuscles on each of them in the same ratio of 2:1. Likewise, when ascending, they increase the speed with which they encounter the corpuscles by the same ratio. Thus a simple one-second pendulum receives weaker impulses from the fluid than does a half-second pendulum. This difference is proportional to the difference in the vertical speeds of the two pendulums. Since one of the pendulums' vertical speeds is twice that of the other, their difference is equal to the lesser of the speeds; that is to say, to the vertical speed of the half-second pendulum. This speed can be estimated at $3/10,800^{\text{ths}}$ that of a body falling freely after one second (a little more than 5 fathoms per second). This is $1/104^{\text{th}}$ part of that of the moon in its orbit (522 fathoms per second). Thus, the difference in the vertical speeds is about $1/86,400^{\text{th}}$ of the speed of the moon. If, then, the speed of the gravific fluid is equal to that of this satellite, and the long pendulum is exactly four times the length of the shorter one, the long one will oscillate more slowly by $1/86,400^{\text{th}}$ of the result given by its theoretical comparison with the shorter one: that is, while the short pendulum makes 172,800 oscillations, the long one will not make 86,400 but only 86,399. However, no such difference in their oscillatory periods has ever been noted over the period of 24 hours, not even when two even more unequal pendulums were compared. Therefore, the fluid that produces weight moves faster than the moon.

18. There is reason to believe that the speed of the corpuscles is much higher, greatly exceeding the speed of light. Admitting this result, and combining it with the duration of the universe, will give an idea of how great the ultramundane sky must be; that is to say, the space occupied by the ultramundane corpuscles.

Article 3

Some Propositions Related to the Subject of this Chapter

19. Theorem 1. The mean thickness of a globe is $2/3$ of its diameter.

20. Theorem 2. When a homogeneous stream of corpuscles meets a sphere, the pressure it exerts perpendicularly to the sphere is $2/3$ that which would be experienced on the base of a circumscribed cylinder with its axis parallel to the stream.

Corollary: This ratio of 2 to 3 will therefore hold good for a plane surface of any size.

21. Theorem 3. A speed equal to that of some celestial bodies would be enough for the gravific corpuscles to conceal a lack of uniformity in the acceleration of free or suspended massive bodies, a deficiency caused by the fact that the relative speed of the massive body and the corpuscles impinging on it is less at the end of a fall than it was at the beginning.

22. Theorem 4. If gravity is caused by the pressure of a fluid moving as fast as Mercury, a one-second pendulum would make one less swing a day than if the fluid's speed were infinite.

23. Theorem 5. If the gravitation of the earth towards the sun is due to the pressure of a fluid moving uniformly in all directions (isotropically) except where it is partially impeded by massive bodies, the fluid must necessarily (and it will suffice if it does) move a hundred-thousand times faster than light. **N.B.** This theorem will be proven in Chapter Four, which deals with the permeability of some terrestrial and celestial bodies (# 36).

24. When a body, impacted by the gravific corpuscles, is first released from its constraints and able to move in response to the gravific pressure, it responds as a whole unit. This is a measure of the speed, v , of the corpuscles. Once the body is free to fall, the pressure decreases imperceptibly and so does its acceleration by a factor of $1/c$. Determine v in terms of c .

Solution: The decrease of acceleration is to the fluid speed as $(24v+3)/2v^2$. This ratio can be approximated as $c=12/v$.

Chapter Three

Concerning the distribution of ultramundane corpuscles

25. The uniform distribution of the corpuscles is necessary for weight and attraction to act equally in all directions.

As far as a body at rest is concerned, it can be shown that it is enough to have 3.141592 different directions in the ultramundane corpuscles for a massive body not to alter its weight by one part in 500,000. The many questions to which this distribution gives rise is more curious than useful and so we do not think it necessary to consider them.

26. Theorem. In substituting a finite number for an infinite number of uniformly-distributed gravific streams, the error thus introduced can be kept to as small a quantity as desired.

Chapter Four

Concerning the permeability of terrestrial and celestial bodies

Article 1 *Of terrestrial bodies.*

27. Theorem 1. A block of terrestrial matter, of moderate density, intercepts less than the ten-thousandth part of the gravific corpuscles passing through it.

Proof: The same commodity has always appeared to weigh the same amount when assembled in a very thick mass as when spread out in a very thin slab, even when the thickness differs vertically. A difference between these two weights of one part in about 2500 would have been detected, and no doubt some physicists would have heard of it and would not have failed to publish the fact: but no such thing has been published. Therefore, this difference, if there is one, is smaller than one part in 2500. Consequently the inner parts of the commodity in the thick form are as about as freely exposed to the action of the gravific corpuscles as are the inner parts of the thin form, the difference being at most one part in 2500. This shows that for the external or upper parts of the thin slab, the difference in the number of interceptions is less than $1/2500^{\text{th}}$ part of the corpuscles causing the weight of the interior parts. It will be shown below that in cases where such a difference in weight would be noticeable, the mean thickness of the upper parts in the first form would be about four inches more than the mean thickness of the upper parts in the second form. Thus, a block of moderate density four inches thick intercepts less than $1/2500^{\text{th}}$ part of the gravific corpuscles arriving to traverse it; and a one-inch thick slab intercepts less than $1/10,000^{\text{th}}$ part of these corpuscles. Q.E.D.

28. First remark: Here is how the same quantity of a commodity can be arranged in two different ways so that the center of mass of the one is covered by a layer thicker by four inches than the center of mass of the other. The first form would be that of a cube with a side nine inches long, whereas the second form will be that of a square slab one inch thick and 27 inches wide. Since the former has its center of mass at a height of 4.5 inches and the other at 0.5 inch, the difference between them is the requisite four inches.

29. Second remark: Instead of these two forms, it is possible to use the following forms instead. One sphere 13.5 inches in diameter can be compared with 729 spheres each with a diameter of 1.5 inch and arranged in an horizontal square with 27 balls to a side and spanning 40.5 inches. As the mean thickness of a ball is two-thirds of its diameter, the thickness of the layers covering the average parts would be, in the large-ball case, 4.5 inches and in the latter case 0.5 inch as in the previous example.

30. Third remark: I am not unaware that should these experiments be made, quantities ten times or perhaps even a hundred times less could be made perceptible. Thus, even in the most delicate experiments, if no difference in weight were observed between the two forms, it could still be concluded that the interception was ten or a hundred times less: that is to say that a one-inch thick slab would only stop $1/100,000^{\text{th}}$ or $1/1,000,000^{\text{th}}$ part of the gravific corpuscles traversing it. But I do not want to set forth here the consequences of experiments that have most likely already been performed and not deal with experiments yet to be made with results which cannot be foreseen. I would have attempted the latter myself, if experiments of another kind, which I shall discuss shortly, had not already show me that the absorption in question is certainly too small for its effects to be perceptible with the current state of the art.

31. Fourth remark: This absorption seems to me to be about ten times less than that of light through salt water (based on the obscurity said to reign at the bottom of the sea). But many absorptions are known, which are much less than that of light. For example, that of magnetic matter through the crust of the terrestrial globe which is at least some hundreds of leagues thick. That the source of magnetism is that far below the surface may be inferred from the mutual correspondence in direction and inclination of magnetized needles in widely separated countries.

32. Fifth remark: “The extreme smallness of the parts of our fluid matter,” says Huygens, “is again absolutely necessary to explain an observed property of weight, namely, that massive bodies, enclosed on all sides in a vessel of glass, metal, or whatever other material it may be, are found to always have the same weight. Thus the matter, which we have said to be the cause of weight, passes very freely through all the bodies deemed the most solid, and with the same facility as it goes through the air.”⁶ This author goes on to treat the question in detail. But there are reasons for not spending too much time or effort on evaluating the limits of the disproportion that appears to be experimentally evident between the vertical thickness of terrestrial bodies (otherwise equal, homogenous and of equal density) and their weight; namely, that the celestial phenomena prove that this disproportion ought to be imperceptible because one layer of average terrestrial matter a fathom thick does not stop even one part in 126,000,000 of the gravific corpuscles passing through it.

Article 2
Of Celestial Bodies:
*Determination of the Impermeability of the Moon*⁷

33. Let us suppose that the spring tides are to the neap tides in the ration 5:3. Consequently, the part due to the sun is one fourth that due to the moon. As this ratio of 1:4 is composed of the ratio of the masses, and of the inverse-cube ratio of the distances, which is 400:1, (that is to say, 64,000,000:1 upon cubing,) it follows that the ratio of the masses is that of 16,000,000:1; so that the cubic root of this ratio is 252:1.

Thus, the shadowing these two globes have on the gravific fluid currents is of the same ratio, 252:1. Therefore, regardless of the effective permeability of the sun, that of the moon will be at most 1/252th of it. The average thickness of the moon is 91,700,000 inches (two-thirds its diameter which is 7/24th that of the earth, which, in turn, is 6,550,000 fathoms). Therefore, every inch of the thickness of lunar matter intercepts or shadows at most one part in 23,108,400,000 of the currents.

34. Remark. This ratio of 16,000,000:1 is to that of that of 350,000:1 which represents the mass of the sun relative to the earth as 320 is to 7 (or 45+5/7) to one. Modern dynamists claim that the ratio of the mass of the earth to that of the moon is greater than that; for example, 343 to 216 or 512 to 343, giving a denominator greater than 252 and a ratio of 7:6 or 8:7. Thus the value of 252 may be more like to 294 or 288. This, in turn, gives a proportion of 1:26,000,000,000 of 1:27,000,000,000.

35. Theorem. There is no phenomenon requiring that the sun in its mean thickness should not intercept at least 1/125th part of the gravific corpuscles passing through it.

⁶ Huygens. *Discours sur la cause de la pesanteur*.

⁷ The paper containing this derivation is dated 5 March 1778. It is later than the preceding non-dated paper, as can be seen from a note in which the author observes that neither paper goes deeply enough into the question of the permeability of the moon and cannot therefore serve as an introduction to the subject. (Note by publisher.)

Proof. As shown in my letter to Mr. Segner,⁸ the phenomenon requiring the greatest permeability in matter is the imperceptibility of a certain eccentricity in the orbits of the satellites of Jupiter. This is referred to by Newton in the sixth proposition of the third of his principles. As I proved, in the same letter, the impermeability of such a satellite should be a ten thousandth if the gravitational acceleration of Jupiter differed from its satellites by 1:2221. Elsewhere I have shown that in the most favorable position available to observe the eccentricity that it will never exceed 12,892 seconds of arc divided by the difference of the gravitational acceleration of Jupiter normalized to that of its satellites.

Therefore, the impermeability of such a satellite will be 1/10,000th if 12,892/2,221 is below the limit of detection. However, owing to the crudeness of the approximations used to measure the greatest elongations of the satellites, it seems likely that 5".8 is well below the limit of detection. It can thus be assumed that the impermeability of such a satellite is 1:10,000.

It should be noted that at least one of the satellites was held to be as big as the earth in the days when it was believed that the solar parallax was in the range from 9 to 10 seconds of arc. (That is to say that the diameter of the sun was taken to be only 100 times that of the earth.) It has just been discovered that this parallax is incorrect,⁹ and this discovery will make it necessary to alter the ratios involving the sun and the satellites. There is no reason to suppose that the density of the sun exceeds that of one of Jupiter's satellites. Therefore, it can be assumed that the average impermeability of the sun is not less than 1:125.

36. Theorem. If the gravitation between earth and sun is due to pressure from an isotropic fluid, being due to shadowing, this fluid must be (and it is sufficient that it should,) have a velocity a hundred thousand times that of light.

To produce the gravitational force between earth and sun, a particulate fluid must move at least twenty times as fast as light, provided that it is also rare enough to produce no noticeable shrinkage of a body over the period of a year. If, then, of the fluid there is only one part in 25,000,000 which is effective in producing gravitation (regardless of whether only one 200,000th part is directed towards the sun or because the sun, at its maximum thickness, intercepts only 1/125th of the protagonist 1/200,000th part), the truly effective part must make up for its lack of density by its speed. It must, therefore, be 5,000 times faster than a fluid moving at twenty times the speed of light. Q.E.D.

First observation. This is a thousand-million times the speed of the earth in its orbit.

Second observation. When I said that the effective portion for gravity had to become 5,000 times faster, I meant the same for all other proportions as well, but without increasing the resistance to the movement of the particles through the earth.

37. Phenomenon. Existing observations may allow the approximate deduction of the amount of absorption of the ultramundane corpuscles by the sun. Particularly, observations of the relative distances between earth and sun, and between Jupiter and sun when such are most easily observable. A month or two before opposition, the sun should appear more to the east than is predicted in the tables of solar position, while a month or two before opposition, the contrary ought to be the case.

⁸ G. L. Le Sage, in his correspondence, often dealt with questions of this kind; and he refers back to these letters in his notes. But it is not always easy to find the copies he kept. (Publisher's note.)

⁹ It is fairly clear that this has indeed happened. And in a later calculation it will be found that the corrected parallax has been used. This irregularity is due to the recent revision of the latter calculation. (Publisher's note.)

38. Impermeability of Jupiter. The mean thickness of Jupiter, according to a determination by M. Maraldi in the proceedings of the Paris Academy of Sciences for 1706, is 5.1922 times the mean distance from earth to sun.

- a) The periods of earth and Jupiter, namely: 365 days 6 hours, and 4332 days 12 hours respectively, are in a ratio of 1461 to 17,320. Thus their squares are as 2,134,521:300,328,900, or as 1:140.70084.
- b) But the square of the period of Jupiter, according to Kepler's law, when applied to M. Maraldi's distance estimate, would be expressed by the cube of 5.1922, or 139.976.
- c) Therefore, the square of the real period is to the square of that given by Kepler's law as 140.701 is to 139.976, about 583:580, or even as 194:193.
- d) At equal distances, however, the centripetal forces follow the inverse-square ratio of the periods.
- e) Therefore, the real centripetal force for Jupiter is to that obtained from Kepler's law-and consequently that of Newton-as 193:194.
- f) According to my theory, then, this diminution of real force in a more massive body is due to the fact that its interior parts are more shielded and thus more out of reach of the gravific corpuscles than are the parts inside less massive bodies.
- g) Thus, the impermeability of the upper hemispheres of Jupiter and earth, reduced to their respective mean thicknesses, should be in the ratio 1:194.
- h) Admitting the secular parallax of the sun to be about 8.6 seconds of arc, the mass of Jupiter is very close to 300 times that of the earth according to the second corollary of Newton's third proposition of his third book of the Principia.
- i) As the impermeabilities of two similar bodies, relative to their sizes, are in direct proportion to these sizes and their densities, the interception of the terrestrial hemisphere is about 87/188ths that of the hemisphere of Jupiter.
- j) [If there is a j, I do not know. It is not in the translator's notes and I do not have a copy of the original French of this section. - *Ed.*]
- k) The interception of Jupiter's hemisphere is thus approximately 188/101^{ths} of 1/194^{ths}, which is about 9/922^{nds}, or a little less than 1/104th.
- l) The impermeability of the whole globe of Jupiter is about 1/52nd.
- m) The impermeability of the whole terrestrial globe is about 1/225th.
- n) The mean impermeability of the sun is greater than that of Jupiter in proportion to their surface gravities, namely, as 10,000 to 945. It is thus about 10,000: 52 times 943 which is about 10/49^{ths}, or slightly more than one fifth. The impermeability of the sun, corresponding to its diameter, is about 15/49^{ths}, or a little more than three tenths.¹⁰

¹⁰ The whole of this passage, entitled Impermeability of Jupiter Corresponding to its Mean Thickness, up to this point has been drafted by Professor L. L'Huilier from the notes of G. L. Le Sage, with some useful modifications. (Publisher's note.)

Chapter Five

Article 1

Permeability of Composite Bodies

39. Four ways of conceiving the immense rarity of bodies, three of which give rise to very great permeability:

First way. When several large beams, each of the same width and thickness, are laid parallel to each other on a plane and so laid as to be separated from each other by nine times the width of one beam, the quantity of wood they contain is a tenth part of the volume of the whole layer. If, now, a similar layer is placed over the first but with those beams pointing in a different direction preventing them from falling between the first layer; and a third layer on that, then a fourth and so on, the volume of wood will still be one tenth of the total volume. Now suppose that each of these big beams is itself a heap of small sticks arranged in the same way as the beams making up the original heap, so that only the tenth part of each beam is really made of wood, then only the tenth part of the tenth part, that is, one hundredth part of the original heap will actually be wood.

Now further assume that the composition of natural bodies is the same as that of these artificial heaps; but that instead of two successive magnitudes (sizes) of sticks there are three, four, etc., each one subordinated to the other in the same way. It will readily be seen that the matter in these bodies will be only the thousandth part of the volume, then a ten-thousandth part, etc. The number of zeros in the fraction will be precisely the number of magnitudes of size. Hence, after 21 orders, for example, the matter will be a thousand-trillionth of the volume (by a trillionth I mean a million-million-millionth).

Second way. The bodies can be conceived as an assemblage of spheres, each of which is composed of uniformly-sized spheres, each of which, in its turn, is composed of homogeneous spheres, etc. The quantity of matter in a heap of such a large number of spheres arranged in any regular figure is to that of a solid block of the same size as the circumference of a circle is to three times the diagonal of the circumscribed square. (This could easily be seen except that the proof I have made of it is too long to be included herewith.) This makes the ratio about $355/113^{\text{th}}$ to 3 times $577/408^{\text{ths}}$, which is that of 48,280 to 65,201, or about 20 to 27. Likewise, for two orders of spheres, the matter will be $48,280/65,201^{\text{ths}}$ if the $48,280/65,201^{\text{ths}}$ of the apparent volume. If there are three orders, the matter will be $(48,280/65,201)^3$ of the original volume, and so on. Therefore, if there are, say, 23 orders of magnitude, the matter will be $(48,280/65,201)^{23}$ of the volume, that is to say, very nearly $1/1,000^{\text{th}}$; and if the number of orders is seven times 23, that is to say 161, the number of zeros in the denominator of the density fraction will be seven times three, or 21. So the density will be one thousand-trillionth.

Third way. Spheres with a diameter much smaller than the distance between their centers, linked together by rigid wires which are much thinner still.

If, for example, these spheres are equal and arranged in a regular order, and if their mass is $48,280/65,201^{\text{ths}}$ of the same mass plus that of the wires, and if their diameter is a ten-millionth part ($1:10^7$) of the distance between their centers, the mass of the resulting body will be a thousand-trillionth ($1:10^{21}$) of the apparent volume.

Fourth way. The lattice of wires alone could be kept and, instead of making them prismatic, make them cylindrical, (as one would normally conceive of them); supposing them to be formed of a line of equal spheres touching each other so as to simplify the calculations.

If, then, the mass of the composite body were not to be more than a thousand-trillionth part of the apparent volume, the distance between the intersecting points of the rows would have to be about $\sqrt{(10^{21} \times 6 \times 20 / 27)}$ times the diameter of the spheres, which amounts to two-thirds of a hundred-thousand-million.

Thus far the translation which was never finished.