GEOCENTRIC MECHANICS I: Geocentricity and $2w\times\boldsymbol{v} + \dot{\omega}\times\boldsymbol{r} + w\times(w\times\boldsymbol{r})$[1]

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In the title we show the sum of three accelerations. They are called Coriolis, Euler, and Centrifugal, and they arise from a simple formal mathematical derivation. In the literature it is obligatory that the interpretation of these three terms be obscured. Their derivation is an exercise in kinematics and only has applicability when we multiply them by a mass, *m*, converting them to forces, which, moreover, humans can sense. These forces are often called *fictitious*, thus obscuring their interpretation. Far from being fictitious they can frequently be real, and in fact, provide a basis for geocentric dynamics. The Apeiron Press of Montreal, Canada has come close to agreeing with this. They call such mechanics "relational mechanics," but draw short of ever stating the obvious geocentric significance, e.g. see Assis.[2]

The Derivation

First, consider an arbitrary vector **A** which, in absolute space, we denote by $\mathbf{A}_s$. In a rotating coordinate system (which rotates only and does not translate) we denote this same vector as

(1) $\mathbf{A}_s(t) = x_r(t)\,\mathbf{i}(t) + y_r(t)\,\mathbf{j}(t) + z_r(t)\,\mathbf{k}(t)$

where the subscript *r* indicates measurement within the rotating system. Note, $x_r(t)$, $y_r(t)$, $z_r(t)$ are the rectangular coordinates measured along the orthogonal unit vectors $\mathbf{i}(t)$, $\mathbf{j}(t)$, $\mathbf{k}(t)$ where their time dependency indicates that they are rotating. Hence, differentiating with respect to time gives

(2) $d\mathbf{A}_s/dt = (x_r d\mathbf{i}/dt + \mathbf{i}dx_r/dt)+(y_r d\mathbf{j}/dt + \mathbf{j}dy_r/dt)+(z_r d\mathbf{k}/dt + \mathbf{k}dz_r/dt)$.

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[1] The symbols are defined as follows (boldface denotes a vector): w is the angular velocity (speed of rotation), $\dot{\omega}$ is any change or acceleration in the angular velocity, and $\boldsymbol{r}$ is the distance from a reference point. The symbol $\times$ denotes the cross or vector product which is also known as the Cartesian product.

[2] Assis, *Relational Mechanics* Apeiron, 1999, pp. 82-82, 150, 154-157, 182-187, 190-192, 196, 199, 212, 218-222, 236.

Let w be the rotation vector of the moving system, then

$$(3)\quad d\mathbf{A}_s/dt = [x_r(w \times \mathbf{i}) + \mathbf{i}dx_r/dt]_r + [y_r(w \times \mathbf{j}) + \mathbf{j}dy_r/dt] + [z_r(w \times \mathbf{k}) + \mathbf{k}dz_r/dt]$$

where, e.g., $d\mathbf{i}/dt = w \times \mathbf{i}$ is the tangential (and only) motion of **i** due to rotation w.

Collecting terms,

$$(4)\quad d\mathbf{A}_s/dt = [w\times(x_r\,\mathbf{i}) + w\times(y_r\,\mathbf{j}) + w\times(z_r\,\mathbf{k})] + [\mathbf{i}dx_r/dt + \mathbf{j}dy_r/dt + \mathbf{k}dz_r/dt]$$
$$= w \times \mathbf{A}_r + d\mathbf{A}_r/dt.$$

We may provide words for equation (4): if $(d\mathbf{A}/dt)_s$ and $(d\mathbf{A}/dt)_r$ are the measured (i.e. by ranging or surveying) time derivatives in the s-system and r-system, then they are related by

$$(5)\quad (d\mathbf{A}/dt)_s = w \times \mathbf{A}_r + (d\mathbf{A}/dt)_r.$$

Equation (5) does not require that the s-system be inertial, however, our application does.

I find this derivation to be mystical, but much more concrete than what is found in the texts. We now proceed to the derivation of the three inertial forces.

Let **r** and **r´** be displacement vectors in the s-system and r-system and let r be the position of the s-system origin with respect to the s-system, then

$$(6)\quad \mathbf{r} = r + \mathbf{r}'$$

and differentiating both sides,

$$(7)\quad (d\mathbf{r}/dt)_s = (d\,r/dt)_s + (d\mathbf{r}'/dt)_s.$$

And using identity (5),

$$(d\mathbf{r}/dt)_s = (d\,r/dt)_s + [(d\mathbf{r}/dt) + w \times \mathbf{r}']_s$$

The differentiating again and applying (5)

$$(d^2\mathbf{r}/dt^2)_s = (d^2 r/dt^2)_s + (d\mathbf{v}/dt)_s + d/dt(w \times \mathbf{r}')_s$$
$$= (d^2 r/dt^2)_s + [(d\mathbf{v}/dt)_r + w \times \mathbf{v}_r] + (dw/dt)_s \times \mathbf{r}' + w \times (d\mathbf{r}'/dt)_s$$

$$= (d^2 r/dt^2)_s + (d\mathbf{v}/dt) + w \times \mathbf{v}_r + [(dw/dt)_r + w \times w] \times \mathbf{r}'$$
$$w \times [(d\mathbf{r}'/dt) + w \times \mathbf{r}']$$

or

$$(8) \quad \mathbf{a} = \mathbf{A} + \mathbf{a}' + 2\, w \times \mathbf{v}' + dw/dt \times \mathbf{r}' + w \times (w \times \mathbf{r}')$$

where

$\mathbf{a}$, $\mathbf{a}'$ denote acceleration in the s-system and r-system, $d^2\mathbf{r}/dt^2$, $d^2\mathbf{r}'/dt^2$
$\mathbf{v}'$ denotes velocity = $d\mathbf{r}'/dt$,
$\mathbf{A}$ is the acceleration of the origin of the r-system = $d^2 r/dt^2$.

This derivation gives the correct result as is verified by experiment, but it so unintuitive that even Leonard Euler missed it.

The term $-2w\times(d\mathbf{r}'/dt) = -2w\times\mathbf{v}'$ is called the Coriolis acceleration, $-dw/dt \times \mathbf{r}'$ is the Euler acceleration and $-w \times (w \times \mathbf{r}')$ is the centrifugal acceleration. Equation (8) is the subject of this paper. It only has sense when it can be sensed by humans. To do this we, in accord with Newton's second law, multiply both sides by *m*, the mass of a test particle thus converting accelerations to sensible forces.

$$(9) \quad m(d^2\mathbf{r}'/dt^2) = m\boldsymbol{a} - 2mw\times(d\mathbf{r}'/dt)' - m(dw/dt)\times\boldsymbol{r}' - mw\times(w\times\boldsymbol{r}') + \boldsymbol{F}'.$$

Hence we now have Newton's second law for a moving frame where applied forces $\boldsymbol{F}'$ have been included. We assume that $\boldsymbol{F}'$ depends only on relative distances and velocities, such as gravitation and Coulomb forces. Hence

$$(10) \quad \boldsymbol{F} = \boldsymbol{F}'.$$

We may also note that $w' = -w$.

Fictitious Forces

Let the test mass, *m*, not be coupled to the primed system (*i.e.*, moves in the unprimed system). For example, an observer flying a rocked ship. If this pilot measured his accelerations with respect to the moving system he would observe, but not sense, equation (8). In this case the Coriolis, Euler, and centrifugal forces are fictitious forces in that they are not felt and have no material cause. And if *m* had been coupled (attached) to the moving frame, then these forces are real.

Geocentric Mechanics

Let the primed system's origin be moved to coincide with the unprimed system's origin and let w lie along the z-axis and be assigned to have a magnitude of one sidereal rotation, *i.e.*, $23^h\ 56^m$ per rotation of the starry sky. We now let the primed and unprimed systems coincide. We further acknowledge that the Coriolis, Euler, and centrifugal terms are a property of the aether (firmament) that God has established. Our geocentric version of Newton's second law is:

$$(11)\qquad m(d^2\mathbf{r}'/dt^2) = m\boldsymbol{a} - 2m\mathrm{w}\times(d\mathbf{r}'/dt) - m(d\mathrm{w}/dt)\times\boldsymbol{r} - m\mathrm{w}\times(\mathrm{w}\times\boldsymbol{r}) + \boldsymbol{F}$$

where we have dropped the primes and regard $\boldsymbol{r}$ as a displacement measured from the earth's center. We furthermore hold that both $\boldsymbol{a}$ and (dw/dt) are zero or very, very nearly so. $\boldsymbol{a}$ and (dw/dt) may on occasion have small non-zero values to accommodate catastrophes that God has wrought upon the earth. $(d\mathrm{w}/dt)\neq\mathbf{0}$ would be a disturbance of the aether and not necessarily a motion of the earth. $\boldsymbol{a}\neq\mathbf{0}$ could likewise be a perturbation of the aether or a displacement of the earth's position from its established place. In this connection we might consider the following verses:

> Psalm 82:5—…all the foundations of the earth are out of course.
> Isaiah 24:19—…the earth is moved exceedingly.
> Isaiah 24:20—The earth shall reel to and fro like a drunkard, and shall be removed like a cottage;… .
> Joshua 10:12 v.f., Joshua's long day.
> Jeremiah 4:24—I beheld the mountains, and lo, they trembled, and all the hills moved lightly.

There are many such like verses.

Applications

Equation (11) is the basis for geocentric mechanics. As we shall see in the sequel, its solutions yield the stationary satellite, the precession of a gyroscope, the rotation of the plane of the Foucault pendulum, and the path of a falling body.

Let the terms in (11) containing w be denoted by $\boldsymbol{F}_i$, then (6) becomes

$$(12)\qquad m(d^2\mathbf{r}'/dt^2) = \boldsymbol{F}_i + \boldsymbol{F}$$

where $\boldsymbol{a}$ has been ignored. The Copernican equation is

(13) $m(d^2\mathbf{r}'/dt^2) = \boldsymbol{F}$.

The initial conditions for (12) are geocentric whereas for (13) they are measured in some indeterminate inertial space. The solution of (13) must receive a transformation to obtain geocentric coordinates. We denote the solution of (12) by $\boldsymbol{X}_g$ (t) and that of (13) by $\boldsymbol{X}_i$(t) and its transformed version as $\boldsymbol{X}_c$(t). $\boldsymbol{X}_g$(t) and $\boldsymbol{X}_c$(t) are not the same, thus presenting a test between the two.

The Literature

Rotation of coordinates is always a confusing subject in the texts. This is so since the inclusion of geocentricity is forbidden in the literature. However, suggested reading is:

Corben & Stehle, *Classical Mechanics,* 2nd ed., pp. 140-146, Dover, 1950.
Kilmister & Reeve, *Rational Mechanics*, Chapter 3, Elsevier, 1966.
Assis, *Relational Mechanics* Apeiron, 1999.
Flügge, *Principles of Classical Mechanics and Field Theory*, pp. 45, 437-440, 489-490, Springer Verlag, 1960.

In these references, the mathematics is greatly embellished and many interesting identities are derived.

Cause for concern!

A Washington, DC airport ticket agent offers some examples of why our country is in trouble!

I just got off the phone with a freshman Congressman who asked, “How do I know which plane to get on?” I asked him what exactly he meant, to which he replied, “I was told my flight number is 823, but none of these planes have numbers on them.”