

HEAVIER OBJECTS FALL FASTER

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If one impells and object it will orbit about the earth's center. If impelled properly it will not collide with the earth's surface. The resulting path is an ellipse with earth's center at a foci. Isaac Newton derived:

$$P = 2 \pi G^{-1/2} (m + M)^{-1/2} a^{3/2}$$

where

P = the orbital period, *i.e.*, time to complete on orbital revolution

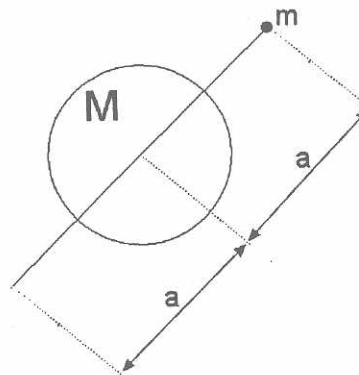
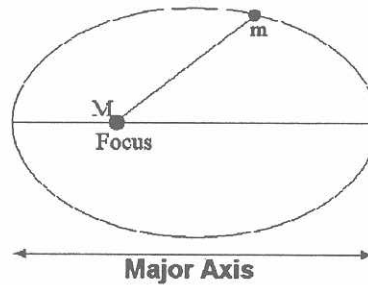
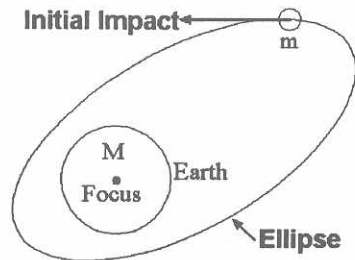
G = constant of gravity

m = mass of object

M = mass of central body, *e.g.*, the earth

a = length of ellipse's major axis.

A special case is where the object is not released with a velocity but is dropped so that the ellipse is reduced to a straight line. In this case (a hole through the earth) the object would oscillate between its initial distance, a , above the earth's surface and its antipodal position, also, a distance, a above the earth's surface.



If one drops masses m_1 and m_2 from the same distance, a , from the earth center then the ration of their corresponding periods is:

$$\begin{aligned} \frac{P_1}{P_2} &= \frac{2 \pi G^{-1/2} (m_1 + M)^{-1/2} a^{3/2}}{2 \pi G^{-1/2} (m_2 + M)^{-1/2} a^{3/2}} \\ &= [(m_2 + M) / (m_1 + M)]^{1/2} \\ &= [(1 + m_2/M) / (1 + m_1/M)]^{1/2}. \end{aligned}$$

As an example, let m_1 be $2/100^{\text{th}}$ s the mass of the earth and m_2 be $1/100^{\text{th}}$ (the moon is $1/81^{\text{st}}$), then:

$$P_1/P_2 = (1.01/1.02)^{1/2} = 0.9951,$$

that is, the small mass's period is larger.